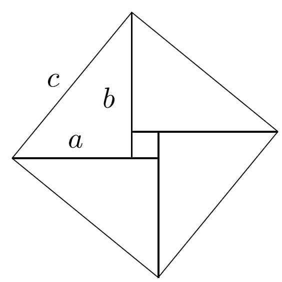
MONT 109N – Mathematics Across Cultures
Discussion/Problem Set 4 – Chinese Mathematics
March 30, 2011

Background and Assignment

In this assignment, we want to look at a sample of Chinese mathematics starting from the time of the *Zhou Bi* (which was perhaps collected in written form as early as 500 B.C.E.) almost to the beginning of the Ming dynasty. Individual writeups are due in class on Monday, April 4.

Questions

A. The *Zhou Bi* contained a diagram like the following one, including four right triangles arranged around a small square to make a larger square, and illustrating a relationship the Chinese knew as the *gou gu* theorem.



(The labels a, b, c are in our Roman alphabet, of course. The original Chinese diagram had Chinese characters, and a square grid overlaid on this picture.)

1) What do *we* call the *gou gu* theorem?

2) How does this diagram essentially furnish a *proof* of that theorem? (Note: It is thought by many historians that the mathematics presented in the *Zhou Bi* goes back to the time of the Greek "Dark Age" between the end of the Bronze Age and the start of the Archaic Period.)

B. The *Jiu Zhang* (The "Nine Chapters in Mathematical Arts") from the Han period (200 B.C.E. to 222 C.E.) contained problems similar to the following one: *Three sheaves of a good crop and 2 sheaves of a bad crop are sold for 39 dou (a unit of money). Two sheaves of a good crop and one sheaf of a bad crop are sold for 25 dou. What are the prices for a sheaf of the good crop and the bad crop?*

1) Write the given information as a pair of equations for x = price per sheaf of the good crop and y = price for a sheaf of the bad crop. Solve your equations by modern algebraic methods.

2) The solution method for this type of problem described in the *Jiu Zhang* worked like this. (They used the *heng/zong* counting rod numerals, but we will use our Hindu-Arabic numerals for simplicity.) Lay out a rectangular array of numbers representing the given information like this:

$$\begin{matrix} 3 & 2 & 39 \\ 2 & 1 & 25 \end{matrix}$$

Then, multiply every element on the second row by 2 and subtract from the corresponding entry on the first row to get a new array:

$$\begin{array}{ccc} -1 & 0 & -11 \\ 2 & 1 & 25 \end{array}$$

Multiply every entry on the first row by 2 and add to the corresponding entry on the second row:

$$\begin{array}{ccc} -1 & 0 & -11 \\ 0 & 1 & 3 \end{array}$$

Explain how and why this process gives the solution you found in part (1). *Note:* Over time, by the Song and Ming dynasties, the Chinese were able to apply similar methods to much larger systems of equations (examples exist of up to 10 equations in 10 unknowns). The algebra is more tedious but the idea is essentially the same – represent the equations in a rectangular array like this, and combine rows in a systematic way to make enough zeroes in the table so that you can "read off" the solutions. (This was only rediscovered in Europe around the beginning of the 19th century!)

3) Have you seen an equivalent way to solve systems of linear equations like this in an algebra class? Note: I believe the latest standards for 10th grade mathematics in Massachusetts include this topic (!)

C. Another problem from the *Jiu Zhang* (and repeated later in a book of Yang Hui, 1261 C.E.). – *There is a bamboo 10 chang (a unit of length) high. The bamboo is broken so the upper end upper end reaches the ground 3 chang from the stem. Find the height of the break.* See the top illustration on the last page for the way the problem appeared in Yang Hui.

D. (Magic squares) The Chinese were very taken with *magic squares* and similar arrangements of numbers. The following method was learned by a French envoy in present-day Thailand in 1687 C.E., but it is thought that the method was developed first in China. A *normal nth order magic square* is an $n \times n$ table consisting of the numbers $1, 2, 3, \ldots, n^2$ arranged so that the sum of the elements in each row and each column of the table is the same number (which must be $n(n^2+1)/2$ in this case). Here is an example of the method using $n = 5$. We first draw a square array of 25 boxes in 5 rows and 5 columns. An additional row is placed along the top and the right side. The square at the top right of the extended table is not used. Starting with $n = 1$ in the middle square on the top row of the 5, put the numbers $1, 2, 3, \ldots, 25$ in order going one step to the right and one up each time.

- If that "move" takes you to the row on the top, "wrap back" to the bottom row and continue from there.
- If the "move" takes you to the column on the right, "wrap back" to the left-most column.
- If the move runs into a previously filled square (or past the upper right corner), drop down one square instead and continue from there. (The X on the top row is a reminder not to fill that space.)

This gives the following table:

	18	25	2	9	X
17	24	1	8	15	17
23	5	7	14	16	23
4	6	13	20	22	4
10	12	19	21	3	10
11	18	25	2	9	

The magic square is then obtained by deleting the “extra” top and right entries:

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

1. Check that that this is indeed a normal magic square of order 5.
2. The same method can be used to produce a normal magic square of order n for any odd n. Use it to produce a normal magic square of order 7.
3. (Extra Credit) Can you *prove* why this method works?

E. The diagram on the bottom of the last page appeared in the *Si Yuan Yu Jian* of Zhu Shijie (from 1303 C.E.) Each circle contains a number written in *heng/zong* form. The small circles are zeroes (a late development in Chinese mathematics that can be traced to contacts with India).

1) Transcribe all of the numbers into our Hindu-Arabic numerals. (Note: Some of the orientations of the *hong/zeng* symbols, and the alternation between *hongs* and *zengs*, are slightly different from the description in Joseph’s book; try to figure them out from “internal evidence”)
2) What is this table (what are the numbers)?
3) What European mathematician/philosopher is credited with finding this pattern in histories of mathematics in Europe and when did he live? Also, do you know why was he interested in these numbers?