

MONT 109N – Mathematics Across Cultures
Discussion/Problem Set 1 – Ancient Mayan mathematics from the *Dresden Codex*
February 4, 2011

Background

The ancient Mayans used a fully positional *vigesimal*, or base-20, number system. They wrote numbers in an extremely economical way using just three different symbols – a round dot for 1, a longer bar for 5, and a “shell-shaped” symbol for 0.

Today, we want to look at pages from one of the four surviving ancient Mayan *codices* – folding books made from fig tree bark, with writing created by brush and ink on a surface prepared with a kind of plaster. The one we will look at is called the *Dresden Codex*. See the photocopy of three of the pages attached to this sheet. Part of what we want to do in this assignment is to decipher and try to interpret a portion of the contents.

This precious record of the ancient Mayan past was created some time after the end of the Classic period, likely copying an earlier source. It was one of the few Mayan books to escape being burned by the Spanish during the period they ruled this part of Central America. A major goal of Spanish colonial rule was to stamp out the cultures of the indigenous peoples in their territories while converting them to Christianity.

The codex is kept in a museum in the German city of Dresden, hence the name. (By the way, the damage across the top of the pages is not ancient – it was caused by water following the fire-bombing of Dresden by the Allies in WWII in February 1945. Amazingly enough, the Germans cared enough about preserving this artifact to save it even though most of the city of Dresden was destroyed and perhaps 25,000 civilians lost their lives in these attacks.)

Take a look at the contents of these pages to start, and note that a great proportion of the contents consists of numbers(!) How should we read them and what do they mean, though?

We are going to concentrate on the material on the *lower half* of the first (*left-hand*) page.

Questions – Note: You will start work on the following questions in (new) discussion groups in class on February 4. The assignment will consist, though, of individual writeups, due in class on Friday, February 11.

- A) Let’s start with the numbers below the two rows of other glyphs. Let’s make the assumption that each is a three-digit base-20 number, with the digits stacked *vertically*, knowing that’s the way the Mayans generally did things. (The other possibility, that the digits are from base-20 numbers listed horizontally leads to improbably huge numbers, so we can reject that possibility, even though other known Mayan texts did deal with truly huge numbers.) But of course, there is a remaining question – *which is the ones digit and which is the $20^2 = 400$ ’s digit?* Do the numbers read “up” or “down?” Of course, the number in the first column is not much help here – it reads *the same* “up” or “down(!)” But, for instance, is the second from the left to be read

as

$$15 + 7 \times 20 + 11 \times 20^2 = 4555,$$

or

$$11 + 7 \times 20 + 15 \times 20^2 = 6151?$$

Try reading the numbers *both* ways and see whether one of them looks more likely to you, based on some order or rudimentary pattern you might observe. Explain how you are deciding which alternative makes more sense.

- B) One way to explore a series of numbers is to combine or work with them arithmetically in various ways. A very useful thing to do with a list of numbers in a lot of situations is to find *the differences* between successive pairs in the list. What do you get if you do that here? What could these mean? Well, the Mayanists who deciphered the Dresden Codex noticed something interesting. As we know, the *lunar month* (the time from say one new moon to the next new moon is approximately 29.5 days). How are the number 177 that appears among the differences and 29.5 related?
- C) The successive differences from part B do not all work out to be whole number multiples of 29.5, though. What's going on? To get a glimpse of something new, look at the numbers in the *last two rows at the bottom of the page*. Decipher them. There should be a tantalizing but incomplete overlap between these numbers and the successive differences from part B.
- D) To explain the discrepancies, we need to make one further "correction." It is known that the ancient Mayans used a *slightly odd system* for recording *numbers of days in calendar calculations*(!) In the notation for numbers of days, instead of a pure base 20 system, a modified or *mixed* system was used in which the digits were numbers of 1's, 20's, $18 \times 20 = 360$'s, 18×20^2 's, etc. This system is probably best explained by the fact that $18 \times 20 = 360$ is closer to the number of days in a solar year than $20^2 = 400$ is(!) Since we think that the numbers here might have something to do with numbers of days, let's go back and recalculate the numbers using this "modified" base-20 system. For instance, the number in the second column might "really" be

$$11 + 7 \times 20 + 15 \times 18 \times 20 = 5551.$$

Recompute all the numbers this way and find the new successive differences. You should see a nice agreement now with the last rows of this page of the codex.

- E) The glyphs on the rows *in between* the rows of numbers from the previous parts are names of particular days in the Sacred Round or "tzol'kin" portion of the Mayan calendrical system we will look at in detail next week. Here's how they translate top to bottom within each column:

Column 1: 4 Ik, 5 Akbal, 6 Kan.

Column 2: 12 Cauac, 13 Ahau, 1 Imix.

Column 3: 7 Cib, 8 Caban, 9 Eznab.

Column 4: 2 Ben, 3 Ix, 4 Men.

Column 5: 10 Oc, 11 Chuen, 12 Eb.

Column 6: 2 Eznab, 3 Cauac, 4 Ahau.

What is the pattern in each column? How are the two dates on the “ends” related to the one in the middle?

- F) *What does the table mean?* The page we looked at is part of a section of the table that is thought to deal with a particular astronomical phenomenon that has interested people of many different cultures. Each of the “middle” dates here is correlated with a Long Count date on which it is known that a solar or lunar *eclipse* (in most cases partial, not total) occurred in the years 770 - 772 C.E. that would have been visible in the Mayan lands.

Solar eclipses occur at a new moon, when the moon is between the earth and sun and blocks out the sun’s light for a certain period. Lunar eclipses occur similarly at a full moon when the earth is between the sun and the moon and appears to block out the moon’s light. If the moon’s orbit was not inclined slightly to the plane of the earth’s orbit about the sun (called the *ecliptic* in astronomy!) then eclipses would occur every month (and would probably have been much less alarming to humans historically!). But because of the inclination of the moon’s orbit, eclipses can only happen when the moon is crossing the ecliptic, and this happens twice in each lunation, at the “ascending node” and “descending node.” Now, if the moon is not “lined up” with the sun and the earth at that time, there is no eclipse. However, if the moon is near the ascending node at the time of an eclipse, it will be near the descending node about half a year later. There is *a good chance that an eclipse will occur again* at the nearest new or full moon. This will usually be six lunar months, or approximately 177 days, after the first eclipse (this can be closer to 178 days in some cases). Several eclipses may follow at 177- (or 178-) day intervals. The lunar month is not exactly 29.5 days long, though, so eventually the differences build up enough and another eclipse in the series cannot occur. At the time of the last eclipse in a series, though, the closest new moon to the node may arrive after five lunar months, approximately 148 days. (It is important to realize that the Earth is rotating while all this is going on, so the eclipses in a series like this might not all be visible from the same location on the Earth.)

Based on this astronomical information, an astronomer named *Martin Meinshausen* came up with an interpretation of what this table in the Dresden codex probably was for the Mayans in 1913. By thinking about this information and your transcriptions of the numbers from the codex page carefully, you should be able to see what his interpretation was. Write a paragraph describing *Meinshausen’s interpretation of what this part of the Dresden Codex is about*. Start with a clearly stated topic sentence, then explain how the calculations you did in the previous parts and the given information about eclipses support your claim.