MONT 108N – Mathematics Through Time
Review/Practice Questions for Midterm Exam
October 22, 2010

General Information

The following are some practice questions indicating the types of mathematical questions I might ask on the midterm exam. Recall that these will account for about 60% of the total points. The remainder of the points will be distributed between short answer and/or multiple choice questions dealing with some of the historical background, and a short essay as described on the information sheet. There are answers for these questions posted on the course home page, but I think it will be best to work through complete solutions and then check your work – don't just look at the answer and say "yeah, that makes sense." You need to be able to do this for yourself with questions you are seeing for the first time.

Disclaimer

These practice problems show some of the range of topics that will be included and the approximate level of difficulty of the exam questions. The actual exam questions may look different and combine topics we have discussed in different ways.

Sample Questions

I.
A) Solve the equation $x^2 - 7x + 6 = 0$ by factoring the quadratic.
B) Solve the equation $x^2 - 7x + 6 = 0$ by means of the quadratic formula and show that the answers you obtain are the same as in part A.
C) Derive the quadratic formula for a general quadratic equation $ax^2 + bx + c = 0$ by the technique of completing the square.

II.
A) Express in base 10: $(101101)_2$.
B) Express in base 10: $(10 : 23 : 24)_{60}$.
C) Express in base 8: $(156)_{10}$.
D) Express in base 60: $(2691)_{10}$.

III. The following two questions are typical of the geometry problems from the Moscow mathematical papyrus. Solve them using modern methods.

A) The area of a rectangle is 48 and the width is 3/4 the length. What are the dimensions?
B) One leg of a right triangle is 5/2 times the length of the other. The area is 20. What are the dimensions?

IV. Compute "the Egyptian way:"

A) 56×125
B) $1970 \div 35$ (find the quotient and the remainder)

V. A Babylonian problem asks for the side of a square if the area of the square, minus the side is the base 60 number $(14 : 30)_{60}$. The tablet says to do this to solve the problem (all numbers in base 60, of course!): "Take half of 1, which is 0.30, square that to get 0.15, add the 14 : 30 to get 14 : 30.15. The last number is the square of 29.30. Now add the 0.30 to get 30, which is the side of the square."
A) Solve this problem using modern methods.
B) Is 30 the correct answer?
C) Explain how the Babylonian method of solution is essentially the same as using the quadratic formula(!)

VI. Recall that we said an integer n is *base-60 regular* if $n = 2^a 3^b 5^c$ for some $a, b, c \geq 0$ (that is, if n is divisible only by the primes occuring in the factorization of 60).
A) What are all the $n \leq 50$ that are base-60 regular?
B) Many Babylonian tables giving $n, 1/n$ are known and all of them contain only base-60 regular n's. Why is that true?

Answers

I.

A) $x^2 - 7x + 6 = (x-1)(x-6)$, so $x = 1$ or $x = 6$.

B) By the quadratic formula

$$x = \frac{7 \pm \sqrt{49-24}}{2} = \frac{7 \pm \sqrt{25}}{2} = \frac{7 \pm 5}{2} = 6, 1.$$

C) We have $a \neq 0$, since otherwise the equation is not quadratic. Hence we can factor a out to obtain $a(x^2 + \frac{b}{a}x + \frac{c}{a}) = 0$. Complete the square in the quadratic:

$$\begin{aligned} x^2 + \frac{b}{a}x + \frac{c}{a} &= \left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} \\ &= \left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{2a}. \end{aligned}$$

This equals zero when

$$\begin{aligned} \left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} \\ x + \frac{b}{2a} &= \frac{\pm\sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \end{aligned}$$

II. A) 45 B) 37404 C) $(234)_8$ D) $(44:51)_{60}$

III.

A) Call the length ℓ. Then $w = 3\ell/4$ and $A = 48 = \ell w = \frac{3\ell^2}{4}$. Solving for ℓ, $\ell = 8$ and $w = 6$.

B) Call the length of one leg ℓ. Then the other is $5\ell/2$ and the area is $20 = \frac{1}{2}\ell\frac{5\ell}{2}$. So $\ell^2 = 80/5 = 16$ and $\ell = 4$. The dimensions are $4, 10$.

IV.

A) By repeated doubling, we get $125, 250, 500, 1000, 2000, 4000$ (we can stop there since the next double would be 64×125, but $64 > 56$. Then since $56 = 32 + 16 + 8$,

$$56 \times 125 = 4000 + 2000 + 1000 = 7000$$

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B) By repeated doubling, we get $35, 70, 140, 280, 560, 1120$. The next double is larger than 1970 so we stop there. Then

$$1970 = 1120 + 560 + 280 + 10 = 56 \times 35 + 10$$

The quotient is 56 and the remainder is 10.

V.

A) Say the length of the side is x. Then the condition given says $x^2 - x = 870$, or $x^2 - x - 870 = 0$. By the quadratic formula we get

$$x = \frac{1 \pm \sqrt{1 + 3480}}{2} = 30, -28$$

Only the positive solution is relevant for the geometry, so $x = 30$.

B) Yes!

C) The Babylonian "directions" applied to any equation of the form $x^2 - px = q$ would say to compute

$$x = \sqrt{\left(\frac{p}{2}\right)^2 + q} + \frac{p}{2}$$

This is equivalent to the solution given by the quadratic formula(!)

VI.

A) $2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 27, 30, 32, 36, 40, 45, 48, 50$

B) Since those are the n for which $1/n$ has a *finite* base 60 expansion! (See Discussion 2)