Montserrat 108N – Mathematics Through Time
Paper Assignment 3

General Information

Your third larger-scale formal writing assignment for the course will be an essay on one of the topics described below. Your paper should be prepared using MS Word or equivalent software, *approximately 5 double-spaced pages in length* (but don't worry if you go a bit over or are a bit short – the length is not the main point). You will be saving the paper as a .doc file and submitting it by email to me by email to:

`little@mathcs.holycross.edu`

The due date/time is *5:00 p.m. on Tuesday, December 14.* Note: this is different from what the course schedule on the class web site originally said. This paper will be the final assignment for the semester and will (in effect) replace the final examination originally planned.

Description

Unlike the first two essays this semester, this one *will be* primarily a "research paper." The idea will be to read the suggested sources carefully to determine answers for the questions posed in the paper topic you choose, then write up your findings in your own words showing where in the sources you found the facts and ideas you are presenting

If you use quotations or other information, you should:

1. Identify direct quotations with quotes and a note in parentheses giving information where the quote can be found within your source,
2. List all your sources in a References (Bibliography) section at the end of your paper:
   - For books, give the author, the title, the publisher and year of publication.
   - For magazine or newspaper articles, give the author, the title of the magazine or newspaper, the date of publication, and the starting and final page of the specific article.
   - For web sites, give an author (if you can determine that), the full URL (web address) and the date you accessed it (since information on the web often changes!)

A Suggestion

I will be happy to discuss (or read a first draft of) your paper and give you some preliminary comments by email. Or, you can come by my office hours if you want to "run your paper by me." Alternatively, I think you may find it very helpful to have a first draft of your paper read by another student in the class. I can set up "reading pairs" if you are interested.

Evaluation

I will provide written comments on your work, and assign two grades for each paper – one for how well your conclusions are presented and supported (in other words, for how convincing your arguments are), the other for how well your writing follows the standards for formal written English. (For instance, how well is the paper subdivided into paragraphs each addressing a particular item in your overall argument? Are the paragraphs arranged in a logical, recognizable sequence? Are the sentences within each paragraph ordered well? Are they grammatically correct? Are there awkward sentences? Are there overly flowery, overly colloquial, or incorrectly-used words or phrases? Is punctuation used correctly? Are there spelling and/or typographical errors?)

Topic 1 – The Final Word on Plimpton 322?

Recall that earlier in the semester we discussed several different interpretations of the Old Babylonian mathematical tablet called "Plimpton 322." In recent years, primarily through the work of a historian of ancient Mesopotamia named Eleanor Robson, another interpretation besides the ones we discussed has become arguably the most convincing one. For this paper topic, you would read a summary of this work that Robson wrote for a mathematical audience in the *American Mathematical Monthly* – follow the link from the course homepage to download a copy. This will be your primary new source (in addition to the class notes and Rudman's book *The Babylonian Theorem*). You might also look at the article "Sherlock Holmes in Babylon" by R. C. Buck listed in the biblography of Robson's article to understand some of Robson's criticisms of that style of mathematical history. In your essay, you should be sure to address the following questions:

1. What is Robson's main point about the proper way that mathematical history should be done? Why does she criticize other work on Plimpton 322 on this score?
2. Why specificially does she reject the idea that Plimpton 322 is a sort of trigonometric table?
3. Why does she reject the idea that Plimpton 322 was primarily a number-theoretic investigation of Pythagorean triples along the algebraic lines suggested by Otto Neugebauer?
4. What does she suggest about the *correct* historic interpretation of Plimpton 322? (And how is that connected with other texts we considered earlier like the YBC 6967 tablet?)
5. Are you convinced by her arguments? (Not everyone is.)

Topic 2 – Number Theory in Euclid's **Elements** *and Beyond*

In class we read Book I of the *Elements*, which leads through properties of triangles and parallelograms to an ingenious proof of the Pythagorean theorem. The *Elements* contains much more than this sort of geometry, though. For this topic, you would discuss some of the properties of the whole numbers (or integers) developed in Books VII through IX of the *Elements*. Your primary source here will be the text of those books and the commentaries from David Joyce's web site that we used before (use the link from the course homepage). You may also consult other modern mathematics texts if you like, but that is not necessary. For the purposes of this topic, I would suggest you concentrate on just the following specific topics (among all the topics covered in the propositions in those books):

1. How are Euclid's definitions at the start of Book VII a sort of "marriage" of geometry with properties of numbers. How are they different from the way we usually think of numbers now?

2. Propositions 1 and 2 in Book VII give Euclid's version of integer division and what we now call the *Euclidean algorithm* for finding the greatest common divisor (or as Euclid says, "greatest common measure") of two numbers. How does this work? How would we write this in modern notation? Work out an example (use Euclid's method to find the greatest common divisor of 244 and 128, say) and include it in your paper.

3. Proposition 20 in Book IX is one of the most famous results in the *Elements*, and it is often considered to be one of the most insightful and beautiful proofs in all of the mathematics we know. What does it say, and how does the proof work?

4. Understanding how the prime numbers are distributed among all the positive integers is still a major area of mathematics. What are some of the questions about primes that have not yet been answered?

Topic 3 – The Platonic Solids

In class we read Book I of the *Elements*, which leads through properties of triangles and parallelograms to an ingenious proof of the Pythagorean theorem. The *Elements* contains much more than this, though. For this topic, you would discuss the famous and beautiful solid geometric figures known as the *Platonic solids* studied in Book XIII of the *Elements* – known today as the *tetrahedron*, *cube*, *octahedron*, *dodecahedron*, and *icosahedron*. A primary source here will be the text of Book XIII and the commentaries from David Joyce's web site that we used before (use the link from the course homepage). For some of the other questions below, you will need to identify other sources. I, or our science librarian, Ms. Barbara Merolli, will be happy to help you with this if you need it.

1. The name "Platonic solid" refers to the ancient Greek philosopher Plato, of course. Why was Plato interested in these figures, and what role did he think they played in the mathematical structure of the universe? Was he correct?

2. In the Remark after Proposition 18 of Book XIII, Euclid sketches a proof that these five solids are the only solid figures, all of whose faces are regular (i.e. equilateral and equiangular) plane polygons, *all of the same type*. How does his argument go?

3. There is another argument for the fact that there are only five of these solids that just uses a relation between the numbers of vertices, edges, and faces, discovered by Leonhard Euler (1707-1783). How does that argument go?

4. Where do the Platonic solids appear in nature?
5. What role did the mathematician and astronomer Johannes Kepler think the Platonic solids played in the mathematical structure of the universe? Was he correct?
6. Given the slightly "wacky" ways some very famous philosophers and scientists have thought about the Platonic solids, is it possible that the idea that the universe is designed along mathematical lines is overrated?

Topic 4 – Hypatia of Alexandria

One of the final famous figures associated with the "University" at Alexandria in Egypt was a (female) mathematician and astronomer named Hypatia. During her lifetime, she was a well-known teacher who also apparently helped her father, Theon, prepare some of his works, including possibly a famous edition of the *Elements* of Euclid. However, the main reasons Hypatia is remembered today are probably

- the fact that she is one of the very few female scholars known from antiquity, and
- the nature of her death – she was brutally murdered by a mob in 415 C.E.

For this topic, you would research what is known about her life and death.

1. Who were some of the students she taught and why did they revere her as a teacher?
2. What role did the fact that Hypatia was a follower of the old pagan religion play in her death? Why was the emerging Christian church opposed to her teachings?
3. What was the role of Bishop Cyril of Alexandria in her death?
4. (Optional) There is a recent film called *Agora* that presents the story of Hypatia from a rather opinionated perspective. If you would like to turn your essay into a film review, that would be OK. I can loan you a DVD of this film if you want to pursue this option. If you do, you will need to make your essay more than just a "thumbs up/thumbs down" judgment on the film as a film, though. You will still need to explore what is known about the actual history and think about how well and how accurately the film follows what is known.

Some sources for this topic are on reserve in the Science Library; consult with Ms. Merolli for these.

Topic 5 – Geometry and Philosophy

Starting from the time of the ancient Greeks, Euclidean geometry, and later non-Euclidean geometry as well, have had a tremendous influence in the development of philosophy. In the book *The Mathematical Experience* by Davis and Hersh (on reserve in the Science Library), there is a section called "The Euclid Myth" that summarizes a lot of this in relatively readable form. The idea of this project would be to grapple with some of the issues raised there and to try to understand

- why Euclidean geometry came to be seen as an exact description of the physical universe, and
- how (or whether) the discovery of non-Euclidean geometry has changed that.

For the essay, try to address the following points:

1. What is the point of the story of Socrates questioning the slave boy from the dialogue known as the *Meno*? How did Plato use this in his philosophy?
2. What are the competing schools of *rationalism* and *empiricism* that Davis and Hersh discuss?
3. What were Kant's ideas about geometric knowledge? What is meant by his categories of "synthetic" and "analytic" *a priori* knowledge?
4. How are Kant's ideas viewed today? Does the fact that there are other mathematically consistent geometries besides Euclidean geometry invalidate what he was trying to say?
5. Are there implications here for what mathematics means and what it is good for?

You may also want to read some of the other parts of that chapter in Davis and Hersh to understand some of the issues in the "crisis in foundations" in early 20th century mathematics.