MONT 107N – Understanding Randomness
Group Discussion 4 – Chi-Square Tests for "Goodness of Fit"
April 14, 2010

Background and Goals

To test whether a given box model "fits" a collection of observed data or not, we have introduced the chi-square statistic and hypothesis testing using the chi-square table from page A106. Given a box model with tickets of several types, plus some observed data from a chance process supposedly modeled by the box model:

- The null hypothesis H_0 for a chi-square test is that the box model *is consistent* with the observed data (and any deviations from the expected values are just due to chance variation).
- The alternative hypothesis H_a can be a different possible way to explain what happened with the data, or it could just come down to saying that the box model *does not fit* the data in some way.
- The *test statistic* is computed. For these tests, we use

$$\chi^2 = \text{sum of } \frac{(\text{observed} - \text{expected (from } H_0))^2}{\text{expected}}.$$

  This value is compared with the chi-square table, using "degrees of freedom" equal to the number of possible outcomes (number of terms in the sum for the χ^2 statistic), minus 1.
- The chance that the test statistic would come out that way, or even "more extremely" is the *observed significance level*, or p-value of the test.

Your solutions for the following problems will be due at the end of class on Monday, April 12.

Discussion Questions

A. In class last Friday, we worked out a first example of one of these tests where the box model specified 600 draws with replacement from the box $[1, 2, 3, 4, 5, 6]$, but the observed data for the number of 1's, 2's, 3's, 4's, 5's, and 6's from throwing a die 600 times was:

$$70, 130, 100, 80, 120, 100.$$

Recall that we saw that the observed χ^2-statistic value of 26 or something more extreme would be very unlikely for a fair die.

1. Suppose now we did 60 throws rather than 600 and the data came out:

$$7, 13, 10, 8, 12, 10.$$

Let's think about this case. Before carrying out the χ^2 test, do you think this is less likely, as likely, or more likely if the die is fair than the case we did in class? Explain your reasoning.

2. Carry out the χ^2 test, estimate the p-value from the table, and interpret your result.
3. Now suppose we did $60,000$ throws and the numbers of 1's, 2's, ... , 6's were:

$$10,193 \quad 9,889 \quad 9,993 \quad 9,928 \quad 10,043 \quad 9,954$$

What does your intuition say in this case? Does the χ^2 test confirm that?

B. The International Rice Research Institute (IRRI) in the Phillipines develops new strains of rice which combine desirable properties such as disease resistance and resistance to insects. They use a model for what happens when strains are cross-bred. One project involved raising strains of rice to be resistant to a particular insect pest. The model indicates that the cross-breeding should produce 1/4 strains where all plants of that strain are resistant, 1/2 strains that are mixed, and 1/4 strains that are not at all resistant. Of 374 strains produced, the IRRI found the following data:

Trait	Number of Strains
all plants resistant	97
mixed	184
no plants resistant	93

1. Before carrying out the χ^2 test, do you think the data looks reasonably close to the $1/4, 1/2, 1/4$ fractions? Explain your reasoning.
2. Carry out the χ^2 test, estimate the p-value from the table, and interpret your result.

C. Historically, the percentages of U.S. Caucasians with blood each of the possible blood types have been:

A	B	AB	O
41	10	4	45

In 2008, a random sample of 200 U.S. Caucasians was selected and the number with each blood type was found to be:

A	B	AB	O
89	18	12	81

Is there reason to believe the current percentages are different from the historical percentages?