MONT 107N – Understanding Randomness
Group Discussion 1
January 22, 2010

Background and Goals

We have discussed "box models" as a way to describe chance processes involving repetition. Recall the key components of a box model are:

- the numbers that go into the box
- how many of each kind there are
- how many draws are made from the box and whether the draws are done with or without replacement

Usually, the outcomes from the chance process will correspond to the possible *sums* of the numbers on the tickets that we draw from the box. Today, we want to practice with several box models of this type and start to understand the random processes they describe. You will be handing in one set of solutions from each group no later than Monday, January 25. (Of course if you are done by the end of class, you can hand your solutions in then, too!)

A. Suppose that we do 1000 random draws with replacement from a box containing exactly three tickets with the numbers $2, 5, 8$.

 1. What is the *smallest* sum we could obtain? What is the *largest* possible sum? Explain your answers.
 2. Explain why the sum of the numbers you draw is probably close to

$$1000 \times \text{average of the box} = 5000$$

 (Hint: What are the chances of drawing each different number? About how many times would you expect to draw each in 1000 draws?)

B. Consider the Nevada version of the casino game roulette as described on Wednesday. One bet you can place in roulette is a wager on *four adjoining numbers* on the table – a collection of four numbers like 11,12,14,15 whose squares share a corner in the grid (see diagram on page 282 of the text). A four numbers bet pays 8 to 1.

 1. Set up and describe a box model that is equivalent to betting repeatedly on a particular collection of 4 numbers (specify the features given in the Background and Goals section above).
 2. What are the average and SD of the numbers in your box from part 1?

3. If you draw 100 times from your box as in part 1, explain why the sum of the numbers you draw is probably close to

$$100 \times \text{average of the box}$$

(Hint: What are the chances of drawing each different number? About how many times would you expect to draw each in 100 draws?)

4. The precise version of the "law of averages" will say that if you draw n times from the box, the chance error – the difference between the sum of the numbers you actually draw and $n \times$ average of box – will almost always be at most $2\sqrt{n} \times$ SD of box in absolute value:

$$|\text{chance error}| \leq 2\sqrt{n} \times \text{SD of box.}$$

What does this say about the relative error:

$$\text{chance error}/n$$

as n gets larger and larger?

C. In roulette, you can also place a bet on any one of the columns $1, 4, 7, \ldots, 34$, $2, 5, 8, \ldots, 35$, or $3, 6, 9, \ldots, 38$ (again, see page 282 in the text). A column bet pays 2 to 1.

1. Set up and describe a box model that is equivalent to betting repeatedly on a particular column (again, specify the features given in the Background and Goals section above).
2. What are the average and SD of the numbers in your box from part 1?
3. If you draw 100 times from your box as in part 1, explain why the sum of the numbers you draw is probably close to

$$100 \times \text{average of the box}$$

4. The precise version of the "law of averages" will say that if you draw n times from the box, the chance error – the difference between the sum of the numbers you actually draw and $n \times$ average of box – will almost always be at most $2\sqrt{n} \times$ SD of box in absolute value

$$|\text{chance error}| \leq 2\sqrt{n} \times \text{SD of box.}$$

What does this say about the relative error:

$$\text{chance error}/n$$

as n gets larger and larger?