MONT 107N - Understanding Randomness<br>Some Sample Final Exam Questions<br>April 29, 2010

1. Briefly explain the following chance and statistical concepts.
(a) Central Limit Theorem.
(b) Law of Averages.
(c) $95 \%$ condence interval.
(d) $\chi^{2}$ statistic.
(e) Null hypothesis of a test of significance.
(f) $p$-value of a test of significance.
2. Short Answer:
(a) (Hypothetical) A college has an elective quantitative reasoning course for first-year students. Each year approximately one fifth of the first-year students elect to take this course. The college does a study of the grades of first-year students. The study shows that after the first- year of college, the students who elect to take this math course have an average GPA for their other three courses that is .1 units higher than the average GPA for the other students for all their courses. Based on this data, the Mathematics department argues that this course raises student GPAs and that every student should be required to take it. Does the Mathematics Department have a good argument or is it possibly flawed? If it is a good argument, explain why, if not, how would you correct the study?
(b) Why are we justified in using a normal curve to make estimates about draws from box models regardless of whether or not the data in the box is normally distributed?
(c) In major national polls, which guide politicians and political candidates in their decision making and play a prominent role in the national media, the sample size is usually 1000 or so. Why are samples of this size used? In particular, what are the benefits of using a sample size of 1000 as opposed to smaller sample sizes or larger sample sizes?
(d) If you want to get polling results with comparable accuracy, do you need a bigger sample size if the polling is done in California than you do if the polling is done in Rhode Island? Explain.
(e) True or False and explain: If the $p$-value of a test of significance works out to $p=.05$ then we can say that the null hypothesis was false.
(f) True or False and explain: Increasing the sample size will always decrease the width of a $95 \%$ confidence interval for an average.
3. A gambler is playing roulette. On each play, he bets $\$ 1$ on four adjacent numbers, which pays 8 to 1 . That is, if one of his 4 numbers comes up, he will win $\$ 8$ on a $\$ 1$ bet. Otherwise, he loses $\$ 1$.
(a) Construct a box model for this bet. (Hint: Recall there are a total of 38 slots on a roulette wheel.)
(b) Compute the average and SD of the box.
(c) If the bet is played 50 times, compute the EV and the SE for the bet.
(d) What are the gambler's chances of winning $\$ 40$ or more on 50 spins of the wheel?
4. A CBS News poll released on November 24, 2008 surveyed a random sample of 706 adults nationwide. It reported that $61 \%$ of Americans say it is too soon for movies to be made about the current Iraq war.
(a) What box model would you use to represent this poll?
(b) Use your model to determine a $95 \%$ confidence interval for the percent of Americans who actually feel this way.
(c) True or False and explain: There is a $95 \%$ chance that the actual percent of Americans who feel this way is in the interval you computed in part b.
5. Before a person gives blood, the Red Cross requires that the hemoglobin in their blood measures 12.0 or higher on an electronic scan. There is typically some error in the measurements. (The following is hypothetical.) Suppose that such a device is being calibrated by five readings on a standardized sample known to have a hemoglobin content of 12.0. The five readings are $11.5,11.9,12.0,12.1,12.1$ for an average of 11.92 . Based on these readings, we want to do a test of significance to determine whether the device is calibrated correctly or not.
(a) What box model would you use to model this calibration process?
(b) Formulate a null and alternative hypothesis for the difference between 12.0 and 11.92.
(c) What significance test should you use? Why?
(d) Carry out your test of significance. What do you get for a a test statistic and observed significance level?
(e) Is the device calibrated correctly, or is it biased?
6. (This problem is adapted from "Heart rate in yoga asana practice: A comparison of styles," Journal of Bodywork and Movement Therapies Volume 11, Issue 1, January 2007, Pages 91-95.) Yoga is often recommended for stress relief, yet some of the more fitness-oriented styles of yoga can be vigorous forms of exercise. The purpose of this study was to investigate differences in heart rate during the physical practice of yoga postures, breathing exercises, and relaxation. The study led groups of participants through three different styles of yoga: astanga yoga, hatha yoga, and "gentle" yoga. Participants wore heart rate monitors during the sessions and their heart rates were monitored repeatedly throughout the sessions. Assume there were three independent groups of 50 participants each in the study. Each group was put through a practice session with each style of yoga. The average and standard deviation
for the heart rates (in beats per minute) of the participants for each style of yoga are given below:

| Yoga style | Average heart rate (bpm) | SD |
| :---: | :---: | :---: |
| astanga | 95 | 12.84 |
| hatha | 80 | 9.32 |
| "gentle" | 74 | 7.41 |

The researchers then applied two-sample $z$-tests to each pair of styles and concluded that there may be different fitness benefits for different styles of yoga practice.
(a) Apply a two-sample $z$-test to the astanga and hatha yoga data. What is your $p$-value?
(b) Apply a two-sample $z$-test to the hatha and gentle yoga data. What is this $p$-value?
(c) Were the researchers' conclusions justified? (Note - the higher the average heart rate attained, the higher the fitness benefit.)
7. The December 7, 2007 Gallup Poll repeated a number of questions about how people perceive the relative ability of the Democratic Party and the Republican Party to handle ten important issues. The poll was of adults 18 years old or older. On the issue of health care, the poll asked the following question: Do you think the Republican Party or Democratic Party would do a better job of dealing with each of the following issues and problems? How about health care policy? Here are the responses for more recent poll and the poll of January 2004:

|  | Republicans | Democrats | No difference/no opinion | Total |
| :---: | :---: | :---: | :---: | :---: |
| 2004 Jan. 9-11 | 357 | 578 | 115 | 1050 |
| 2007 Nov. 30-Dec. 2 | 291 | 592 | 120 | 1003 |

(Data was adapted from the Gallup web site.) The natural question to ask is whether or not this represented a real shift in public opinion.
(a) Formulate a null and alternative hypothesis for this question.
(b) What significance test should you use to determine the answer?
(c) Carry out your test of signficance. What do you get for a test statistic and observed significance level?
(d) Did this data represent a real shift in public opinion?

