College of the Holy Cross, Fall Semester 2018
MONT 104N – Modeling the Environment
Final Exam, December 13

Your Name: ___________________________

Instructions Please write your answers in the spaces provided on the following pages, and show work on the test itself. **For possible partial credit, you must show work**. Use the back of the preceding page if you need more space for scratch work.

Please do not write in the space below

Problem	Points/Poss
I	/ 20
II	/ 20
III	/ 15
IV	/ 15
Essay	/ 30
Total	/100

Do all work on these sheets. You may use the back sides if necessary. There are 100 possible points (distributed as indicated in the questions).

Have a peaceful and joyous holiday season!

I. The table below shows estimates collected in the EDGAR database created by the European Commission and the Netherlands Environmental Assessment Agency of the amounts of *carbon dioxide* emissions from burning of fossil fuels in 2015 and 2016 by country. The units are megatonnes = 10^6 metric tonnes. The populations are in units of millions = 10^6 of people:

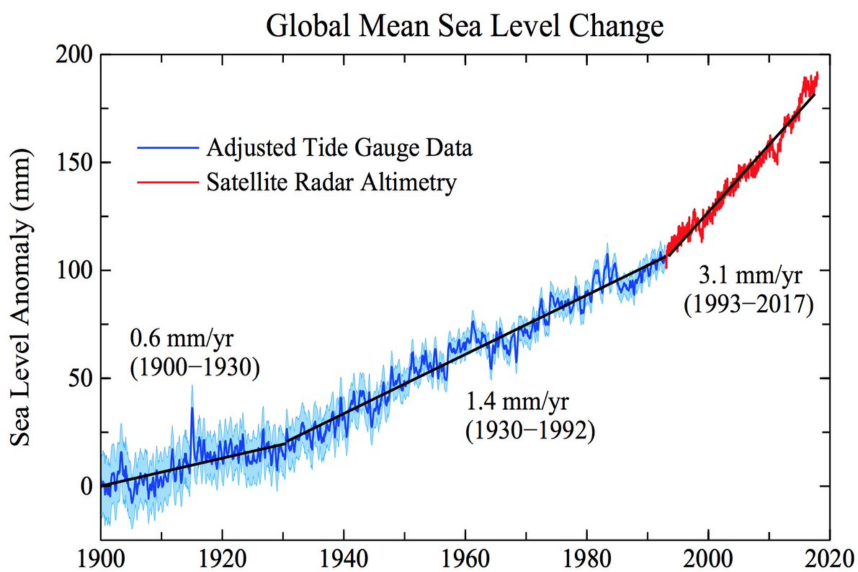
Country	**CO2 2015**	**CO2 2016**	**2015 Population**
China	10.642	10.433	1367
United States	5.712	5.012	321
India	2.455	2.533	1252
Russia	1.761	1.662	142
Japan	1.253	1.240	127
Germany	.778	.776	81
Whole world	36.061	35.753	7256

A. (5) 1 metric tonne is 1000 kg and 1 kg $\doteq$ 2.205 lb. What was the equivalent amount of carbon dioxide emissions for the U.S. in 2015, in units of pounds.

B. (5) Suppose that China's carbon dioxide emissions were decreasing *exponentially*. What would be the exponential model fitting the two data points you have exactly? Take $t = 0$ to correspond to the year 2015. What would your model predict for China's emissions in the year 2020?

C. (10) Now, let's look at this data from another perspective. What are the *per capita* carbon dioxide emissions in units of metric tonnes per person for each of these countries? Construct a chart or charts (your choice of types) showing how the 2015 total emissions and the 2015 emissions per capita compare for these countries. Pay special attention to any changes in the relative orderings when you go from the total emissions to the per capita emissions.

II. The following graph, constructed by Makiko Sato and James Hansen at the Columbia University Climate Science, Awareness and Solutions project, shows the change in global average sea level over the period 1900 to the present. It's also a nice example of how the basic modeling techniques we have discussed can be adapted to deal with dynamic, changing situations.



The vertical axis is the *sea level anomaly*—the *difference* between the observed levels in later years and the average level in 1900.

A. (12) The information included in the graph shows that Sato and Hansen have constructed three separate linear models over three different time periods: one covering the period 1900 - 1930, the second covering the period 1930 - 1992 and the last one covering the period 1993 to 2017. The estimated slopes are given in units of mm/yr. Find the equations of the three linear models (the equations of the three black lines in the graph) for average mean sea level $L(t)$, using $t =$ actual year in each case. Use the values 0 for $L(1900)$, 18 for $L(1930)$, and 104.8 for $L(1993)$.

B. (3) The legend indicates that the first two models were found from adjusted tide gauge data, while the third was obtained from satellite radar measurements. Which of these two methods would you guess is more accurate? How does that relate to the appearance of the graph(s)? Discuss briefly.[1]

[1]Note: The lighter blue region around the blue line plots shows a series of "error bars" indicating a reasonable range of values consistent with the measured data. There's something like that too around the red portion of the plot, but it's much less visible.

III. Suppose that a endangered population of spotted owls in a protected forest is *decreasing* at a net rate of 10% per year from births and deaths, but human-reared owls are being reintroduced into the habitat at a rate of 16 individuals per year.

A. (5) Write a difference equation that models the owl population.

B. (5) Using an initial value $P(0) = 40$, determine the populations in years $1, 2, 3, 4, 5$ according to the model you stated in part A and record the values in the following table (round any decimal values to the nearest whole number)

n	0	1	2	3	4	5
$P(n)$	40					

C. (5) What happens to the population in the long run? Does it tend to a definite value as n increases? If so, what is it? If not, why not?

IV. Answer *any three* of the following briefly. If you submit answers for all four, only the best three will be counted.

A. (5) If you are fitting a *power function* model to a data set (x_i, y_i) "by hand," you would start by transforming the data to a new form (X_i, Y_i). What is that form in terms of the x_i and y_i?. If the best fit regression line for the transformed data is $Y = mX + b$, what is the corresponding power function model? (You may use logarithms to base 10 as we discussed in class.)

B. (5) What difference equation would model a population undergoing logistic growth if the population was growing at about 3% per year when the population is much smaller than the carrying capacity $M = 1000$ of the habitat?

C. (5) Describe the SIR model for infectious disease outbreaks and give the corresponding difference equations.

D. (5) What feature of the solutions of the Lotka-Volterra equations is considered to be a confirmation that this model is capturing an important aspect of real-world predator-prey interactions?

Essay (30)

In general terms, what is a mathematical model? Describe in general terms what they are, how they are constructed, and how they are used. Give examples of three different types of mathematical models we have studied this semester. Even if mathematical models don't capture *every feature* of a real world situation, why is it still important to develop them and understand the information we get from them? For instance, what conclusions about use of natural resources did we derive by looking at logistic models with various types of harvesting in the Chapter Project from Chapter 7? As another example, how are mathematical models important in understanding our choices of which energy sources to use? Why is it important to understand how radioactive substances decay? What type(s) of model(s) that we discussed would apply to describe that process? What are some of the issues involved with using radioactive decay to generate electricity–that is, why is this not a "no-brainer" as a solution to the problem of CO_2 buildup in the atmosphere from fossil fuel burning?