MONT 104N – Modeling the Environment
Solutions for Midterm Exam – November 1, 2019

I. In discussing amounts of water needed for irrigation of farmland, a common unit of volume is the acre-foot. One acre-foot of water is the amount of water necessary to cover a flat field one acre in area to a depth of one foot. Using the information below, answer questions A and B.

$$1 \text{ yard} = 3 \text{ feet}$$
$$1 \text{ acre} = 43560 \text{ square feet}$$
$$1 \text{ meter} \doteq 3.28 \text{ feet}$$

A. (10) How many cubic yards (yd^3) of water are in one acre-foot?
Solution:

$$1 \text{ acre} - \text{foot} = 43560 \text{ ft}^3 = 43560 \text{ ft}^3 \times \frac{1}{3^3} \text{ yd}^3/\text{ft}^3 \doteq 1613.3 \text{ yd}^3$$

B. (10) How many cubic meters (m^3) of water are in one acre-foot?
Solution:

$$1 \text{ acre} - \text{foot} = 43560 \text{ ft}^3 = 43560 \text{ ft}^3 \times \frac{1}{(3.28)^3} \text{ m}^3/\text{ft}^3 \doteq 1234.4 \text{ m}^3$$

II. The following table gives amounts (in tons) of materials of different types recycled in Washington State in two years, 1986 and 1998.

Category	**1986**	**1998**
Paper	391994	821994
Metal	9528	318710
Plastic	349	9871
Glass	48013	113338
Organics + Others	352	903466

A. (15) Construct a chart *for 1998* showing the percentages of that year's total accounted for by each of these types of materials. Any reasonable type of chart is OK.

Solution: We first need to compute the total amount of all types for 1998, then compute the percents of each type of recyclable. Here is a pie chart (computed with

Google Sheets). (If you used a pie chart, it doesn't need to look this good, of course!)

B. (5) What percent of the total *for 1986* was accounted for by Paper and Plastic together?

Solution: The total for 1986 was 450236 tons, while the sum of the Paper and Plastic categories was 392343 tons. The percentage accounted for by Paper and Plastic together was

$$\frac{392343}{450236} \times 100\% \doteq 87.1\%$$

III.

A. (10) In 2012, there were about 5.3×10^4 *alternative fuel vehicles* (powered by electricity, ethanol, natural gas, etc.) sold in the U.S. By 2016, that number had risen to 1.6×10^5. Construct a linear model for AFV = number of alternate fuel vehicles, as a function of t = years since 2012.

Solution: The slope is

$$m = \frac{1.6 \times 10^5 - 5.3 \times 10^4}{2016 - 2012} = 26750$$

(vehicles per year). The equation of the linear model is

$$\text{AFV} = 26750t + 5.3 \times 10^4$$

B. (5) What does your model predict about the number of AFV sold in 2017? The actual number was 1.95×10^5 vehicles. What is the percentage difference between the predicted number (the "comparison") and the actual number (the "reference")?

Solution: The model prediction is

$$AFV = 26750 \cdot 5 + 5.3 \times 10^4 = 186750 \doteq 1.9 \times 10^5$$

The percent difference between this and the actual number is

$$\frac{1.9 \times 10^5 - 1.95 \times 10^5}{1.95 \times 10^5} \times 100\% \doteq -2.6\%$$

(In other words, the model prediction is about 2.6% too small. If you use the value 186750 instead, then the percent difference is about -4.4%. That is OK too.)

IV. Wind power has emerged as one of the faster-growing methods of electricity generation in recent years. In 2016, the generating power of wind turbines installed around the world was about 301 gigawatts and it was increasing at about 33.2% per year.

A. (10) Using this information, construct an exponential model for WP = wind power generation as a function of t = years after 2016.

Solution: The model is

$$\begin{aligned} WP &= (301) \cdot \left(1 + \frac{33.2}{100}\right)^t \\ &= (301) \cdot (1.332)^t \end{aligned}$$

(the units are gigawatts).

B. (5) According to your model, how long will it take for the generating power of wind turbines to reach 3000 gigawatts?

Solution:

$$3000 = (301) \cdot (1.332)^t$$

when

$$t = \frac{\log(9.967)}{\log(1.332)} \doteq 8.02$$

That is, slightly more than 8 years after 2016, so 2024.

Essay (30)

There is an ongoing movement at this point in history (i.e. 2019) that is especially strong in Sweden, where it known as *flygskam*. What does this Swedish word mean in English? What actions are proponents of *flygskam* urging other people not to continue doing, and what are their reasons for doing this? Part of the underlying reason for the existence of this movement is concern about how the actions are influencing greenhouse gas levels in the atmosphere. What proportion of current CO_2 emissions can be attributed to these actions and which people are contributing most to this? While the recommendations of proponents of *flygskam* might make a lot of sense from the environmental point of view, there are also certainly aspects of modern life that would be drastically changed if those recommendations were adopted by everyone. What would we be losing by following those recommendations? Are the environmental benefits great enough to justify the costs to individuals and society of adopting those recommendations?

Model Response: The Swedish word *flygskam* means "flying shaming" or maybe better, "shaming for flying." This refers to the fact that many environmental activists, especially in Sweden, are vocally encouraging people to *cut down on their use of air travel* by shaming them when they travel by air and bragging about using different transportation modes instead. (For instance, this is the reason that Greta Thunberg sailed from Europe to attend the opening of the United Nations session in New York in September this year rather than flying.)

The reason is that air travel is a significant source of CO_2 (and other greenhouse gas) emissions into the upper atmosphere from jet engine exhaust. For instance, it is estimated that air travel accounts for between 2% and 2.5% of all anthropogenic (human-produced) CO_2 at present. The individuals who are contributing to this the most are the "frequent flyers" who use air travel to get to business destinations (and academic meetings–see below), plus people who are flying for tourism. If the demand for this was reduced and people used video conferencing for meetings, or made more use of less carbon-intensive public transportation methods such as passenger rail, then airline schedules could be trimmed (fewer total flights), and total CO_2 emissions would be reduced (at least somewhat).

The biggest thing that we would lose if we seriously curtailed the availability of air travel would be *convenience* in planning trips. Fewer flights means fewer options and probably higher prices. Passenger rail is an attractive option for shorter distances on the same continent, such as the distances between many cities in Europe. However land transportation for distances over about 400 miles is inevitably slower than air travel. For instance, you a direct flight from Boston to Denver (about 2000 miles) takes about 5 hours (and including travel to and from the airports only adds another 2 hours or so). On the other hand, the currently-available rail option (Amtrak) takes about 40 hours (including a layover in Chicago and 2 nights on the train – I know about this because I recently booked a trip to the upcoming Joint Mathematics Meetings in Denver in January). Riding a bus would take even longer and be much less comfortable. Google says you could probably drive the whole way on Interstate highways in about 30 hours, but that time assumes you never have to stop, sleep, or eat, and you never hit any traffic–totally unrealistic!

Perhaps more importantly, it would be much harder to gain the direct face-to-face contact with other people and other cultures that is possible with modern air travel. The

quotation from Mark Twain on the exam review sheet is an eloquent statement about how travel can help drive out narrow-mindedness and foster understanding of other peoples.

Whether the environmental benefits are great enough to justify this loss of convenience of travel is in part a matter of opinion. I'm personally torn about this. The total CO_2 emissions due to aviation are not an especially large part of the total CO_2 emissions and other sources are much more significant. I do try to use rail options whenever I can, but part of that is just that I really enjoy rail travel. There are also times when air travel can be necessary because of time constraints, and auto travel is necessary for "out of the way" destinations.