College of the Holy Cross, Fall Semester 2019
MONT 104N – Modeling the Environment
Final Exam, December 20

Your Name: ____________________________

Instructions Please write your answers in the spaces provided on the following pages, and show work on the test itself. **For possible partial credit, show work even if you cannot completely solve a part of a question**. Use the back of the preceding page if you need more space for scratch work. There are 100 possible points (distributed as indicated in the questions).

Please do not write in the space below

Problem	Points/Poss
I	/ 20
II	/ 20
III	/ 15
IV	/ 15
Essay	/ 30
Total	/100

Have a peaceful and joyous holiday season!

I. The table below shows estimates collected in the EDGAR database created by the European Commission and the Netherlands Environmental Assessment Agency of the amounts of *carbon dioxide* emissions from burning of fossil fuels in 2015 and 2016 by country. The units are megatonnes = 10^6 metric tonnes. The populations are in units of millions = 10^6 of people:

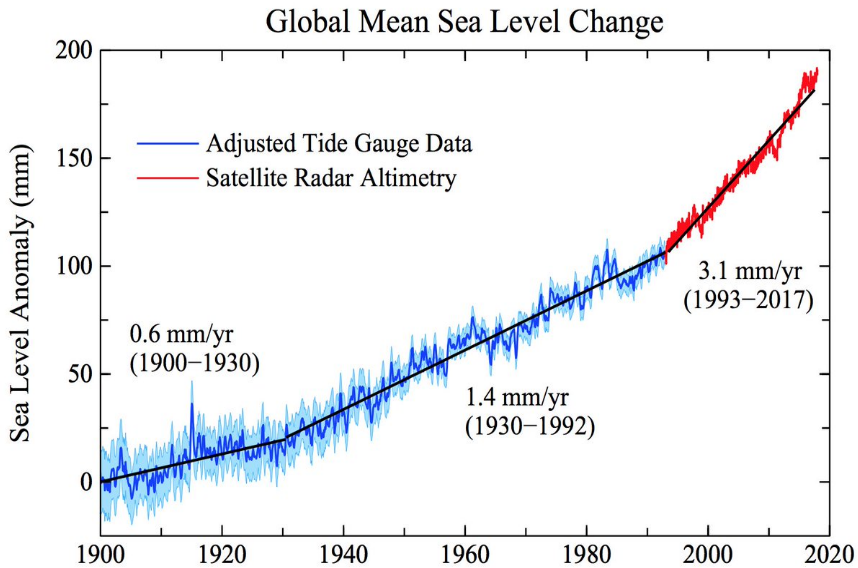
Country	**CO2 2015**	**CO2 2016**	**2015 Population**
China	10.642	10.433	1367
United States	5.712	5.012	321
India	2.455	2.533	1252
Russia	1.761	1.662	142
Japan	1.253	1.240	127
Germany	.778	.776	81
Whole world	36.061	35.753	7256

A. (5) 1 metric tonne is 1000 kg and 1 kg $\doteq$ 2.205 lb. What was the equivalent amount of carbon dioxide emissions for India in 2016, in units of pounds.

B. (5) Suppose that the U.S. carbon dioxide emissions were decreasing *exponentially*. What would be the exponential model fitting the two data points you have exactly? Take $t = 0$ to correspond to the year 2015. What would your model predict for U.S. emissions in the year 2020?

C. (10) Now, let's look at this data from another perspective. Which countries here had *per capita* CO_2 emissions greater than the world average, and which are less than the world per capita figure in 2015?

II. The following graph, constructed by Makiko Sato and James Hansen at the Columbia University Climate Science, Awareness and Solutions project, shows the change in *global average sea level* over the period 1900 to 2017 (i.e. the present). It's also a very nice example of how the basic modeling techniques we have discussed can be adapted to deal with dynamic, changing situations.



The vertical axis is the *sea level anomaly*—the *difference* between the observed levels in later years and the average level in 1900.

A. (9) The information included in the graph shows that Sato and Hansen have constructed three separate linear models over three different time periods: one covering the period 1900 - 1930, the second covering the period 1930 - 1992 and the last one covering the period 1993 to 2017. The estimated slopes are given in units of mm/yr. Find the equations of the three linear models (the equations of the three black lines in the graph) for average mean sea level $L(t)$, using $t =$ actual year in each case. Use the values 0 for $L(1900)$, 18 for $L(1930)$, and 104.8 for $L(1993)$.

B. (3) Why do you suppose Sato and Hansen chose to fit three separate linear models rather than a single exponential or power law model?

C. (3) What is the "take-away" message from this data and this graphic? How would you summarize the conclusion(s) we should draw from it?

III. Suppose that a endangered population of salamanders in a protected wetland is *decreasing* at a net rate of 15% per year from births and deaths, but human-reared salamanders are being reintroduced into the habitat at a rate of 40 individuals per year.

A. (5) Write a difference equation that models the change in the salamander population from each year to the next.

B. (5) Using an initial value $P(0) = 100$, determine the populations in years $1, 2, 3, 4, 5$ according to the model you stated in part A and record the values in the following table (round any decimal values to the nearest whole number)

n	0	1	2	3	4	5
$P(n)$	40					

C. (5) What happens to the population in the long run? Does it tend to a definite value as n increases? If so, what is it? If not, why not?

IV. Answer *any three* of the following briefly. If you submit answers for all four, only the best three will be counted.

A. (5) If you are fitting an *exponential* model to a data set (x_i, y_i) "by hand" (i.e. not using the shortcuts available in a Google spreadsheet) you would start by transforming the data to a new form (X_i, Y_i). What is that form in terms of the x_i and y_i?. If the best fit regression line for the *transformed data* is $Y = mX + b$, what is the corresponding exponential model? (You may use the exponential function with base 10 as we discussed in class, or any other exponential function.)

B. (5) An animal population (in the absence of interactions with humans) would undergo *logistic growth* with a net growth rate 15% per year when the population is much less than the carrying capacity $M = 1000$ of its habitat. Assume in addition that humans are doing constant harvesting of $h = 50$ individuals per year from this population. What difference equation models this situation?

C. (5) How do you determine *equilibrium value(s)* of a first order difference equation $Q(n+1) = AQ(n) + B$ $(A, B$ constant)?

D. (5) What feature of the solutions of the Lotka-Volterra equations is considered to be a confirmation that this model is capturing an important aspect of real-world predator-prey interactions?

Essay (30)

In general terms, what is a mathematical model? Describe in general terms what they are, how they are constructed, and how they are used. Give examples of three different types of mathematical models we have studied this semester. Even if mathematical models don't capture *every feature* of a real world situation, why is it still important to develop them and understand the information we get from them? For instance, what conclusions about use of natural resources did we derive by looking at logistic models with various types of harvesting in the Chapter Project from Chapter 7? As another example, how are mathematical models important in understanding our choices of which energy sources to use? Why is it important to understand how radioactive substances decay? What type(s) of model(s) that we discussed would apply to describe that process? What are some of the issues involved with using radioactive decay to generate electricity–that is, why is this not a "no-brainer" as a solution to the problem of CO_2 buildup in the atmosphere from fossil fuel burning?