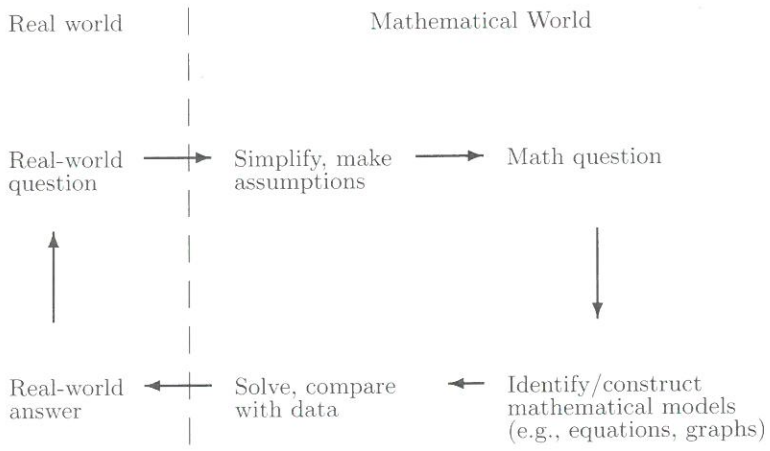

Figure 4.6. The mathematical modeling process

4.4. Mathematical Models

Many of the mathematical topics we will study in this section of this book will be applied to construct *mathematical models* of various real-world and environmental situations. The idea of mathematical modeling is shown in the diagram given in Figure 4.6. This sort of mathematical work is (almost) always motivated by the desire to understand and/or make predictions about the behavior of some real-world system.[4]

The modeling process can be viewed as a sort of "loop" or iteration as shown by the arrows in Figure 4.6. The loop should ideally start with a deep understanding of the real-world question, and for this reason mathematicians doing this sort of work often collaborate with biologists, demographers, climate scientists, and other experts in the applications areas they want to study. Moving into the mathematical world, the underlying real-world question is often reformulated and abstracted as a mathematical question. This often involves making simplifying assumptions or leaving out or ignoring aspects of the real-world system that are thought to be irrelevant (or at least less important) for the problem at hand.

A mathematical model can be an equation, or a graph, or some other mathematical structure that captures some of the features of the real-world system under study. Mathematical techniques make it possible to compute solutions of equations or produce other mathematical results. Is that the end of the story? *Definitely not.* These results from the realm of mathematics must be *tested against*, or compared with, data generated from the real-world system.

- If there *is* sufficiently close agreement or the mathematical results yield good insights about that system, then the model has produced useful information.

[4]Some mathematicians also study mathematics "purely" for its own sake, for the beauty they see in its structures, or for other reasons, without any applications in mind. One of the surprising things we can see from the history of the subject is that even though many mathematical techniques were originally developed without any applications in mind, they have later turned out to be useful for applications even so. A famous essay by the physicist Eugene Wigner called "The unreasonable effectiveness of mathematics in the natural sciences" discusses some of this; see [3].