MONT 104N – Modeling the Environment
Solutions for Mathematical Review Questions from Midterm Exam
October 28, 2019

I. The following table gives amounts of carbon (in units of 10^{15} kilograms) contained in the major "reservoirs" of this element on planet Earth:

Reservoir	**Carbon Content**
Atmosphere	.59
Crust (as fossil fuels)	3.7
Vegetation	2.3
Shallow Ocean	.9
Deep Ocean	37.3

A. What percent of the total carbon present on Earth is contained in each of these reservoirs?

Solution: The sum of these amounts is 44.79×10^{15} kilograms. So, the percentage of the carbon in the deep ocean reservoir, for instance, is

$$\frac{37.3}{44.79} \times 100\% \doteq 83.3\%$$

The other values are computed similarly. Rounding to 1 decimal place,

Reservoir	**Percent of Total Carbon**
Atmosphere	1.3%
Crust (as fossil fuels)	8.3%
Vegetation	5.1%
Shallow Ocean	2.0%
Deep Ocean	83.3%

B. Construct and draw a chart (your choice of type) showing how the total carbon breaks down into these categories.

Solution: Either a pie chart or a bar chart is OK here. If you use a pie chart, the percents above correspond to the fractions of the pie. If you use a bar chart, it would be best to make the vertical axis scale correspond to the actual amounts of carbon (in units of 10^{15} kg), so use something like a vertical scale of 0 to 40. However, a scale where the vertical axis shows the percents is also acceptable.

II. Burning coal provides between 9500 and 14000 BTUs of heat energy per pound. Using the table below, answer questions A and B.

- 1 lb $\doteq$ .454 kg
- 1 BTU $\doteq$ .252 Kilocalorie

A. (8) Express the heat energy from burning one pound of coal as a range of values in calories.

Solution: There are .252 Kilocalories in one BTU. Hence, burning one pound of coal yields between

$$9500 \text{ BTU} \times .252 \,\frac{\text{Kcal}}{\text{BTU}} \doteq 2.39 \times 10^3 \text{ Kcal}$$

and

$$14000 \text{ BTU} \times .252 \,\frac{\text{Kcal}}{\text{BTU}} \doteq 3.53 \times 10^3 \text{ Kcal.}$$

B. (8) What is the range of heat energy values provided by burning 1 kilogram of coal, expressed in BTUs?

Solution: From given information, one pound is .454 kg, so 1 kg $= \frac{1}{.454} \doteq 2.20$ lb. Hence burning 1 kg of coal provides between

$$9500 \text{ BTU/lb} \times 2.20 \text{ lb} = 2.09 \times 10^4 \text{ BTU}$$

and

$$14000 \text{ BTU/lb} \times 2.20 \text{ lb} = 3.08 \times 10^4 \text{ BTU.}$$

C. (4) If the 9500 BTU figure comes from bituminous coal and the 14000 BTU comes from anthracite coal, what is the percentage difference in heat energy between anthracite and bituminous?

Solution: The percentage difference is

$$\frac{14000 - 9500}{9500} \times 100\% \doteq 47.3\%$$

(the anthracite coal generates 47.3% more heat than the bituminous).

III. (15) The Honda Civic comes in a standard (gasoline engine) version and a hybrid (gasoline-electric) version. The standard version has a fuel efficiency of about 34 miles per gallon, while the hybrid version gets about 44 miles per gallon. The hybrid version has a price about $4000 more than the standard. If you bought the hybrid and drive about 10000 miles per year, about how many years will it take for your savings in gasoline costs to make up for the difference in price? Assume that gasoline will average $2.00 per gallon over the life of the vehicle.

Solution: If you drive 10000 miles with the hybrid version, then you use $10000 \times \frac{1}{44} \doteq 227.3$ gallons of gas in one year and you spend $227.3 \times \$3.00 \doteq \$681.90$ on gasoline. With the standard version you use $10000 \times \frac{1}{34} \doteq 294.1$ gallons of gasoline and you spend $882.30.

The difference is \$200.40. To make up the \$4000 greater cost of the hybrid model, you would need to keep the hybrid for about $\frac{4000}{200.40} \doteq 20$ years. (Comment: These are pretty realistic numbers, and this is one reason why hybrids are still seen as less attractive options by many people.)

IV. Consider the Excel scatter plot given in the exam. Place a check next to the best responses to the questions below.

A. (5) The least squares regression line for this data set would have a slope m about:
$m = -.1$ ____*X*____
$m = .3$ ____________
$m = 1$ ____________

The best answer is about $m = -.1$ since the slope definitely should be negative and the velocity is decreasing by roughly 1.4 feet per second over the roughly 11 feet change in depth. The actual slope is $m \doteq -0.12$.

B. (5) Based on the scatter plot, the R^2 statistic is probably:
between .8 and 1 ____*X*____
between .4 and .6 ____________
between .2 and .4 ____________
between .0 and .2 ____________

The best answer is $.8 < R^2 < 1$ since the data is relatively linear (so R^2 will be close to 1). The actual value is $R^2 \doteq .88$.

V. Wind power has emerged as one of the faster-growing methods of electricity generation in recent years. In 2016, the generating power of wind turbines installed around the world was about 301 gigawatts and it was increasing at about 33.2% per year.

A. The typical English unit of power is the horsepower. 1 horsepower $= 7.457 \times 10^{-7}$ gigawatts. Convert 301 gigawatts to the equivalent number of horsepower.

A. *Answer:*

$$301 \text{ Gw} = 301\text{Gw} \cdot \frac{1}{7.457 \times 10^{-7}} \text{ horsepower/Gw} \doteq 4.036 \times 10^{8} \text{ horsepower.}$$

B. Construct an exponential model for WP = wind power generation as a function of t = years after 2016. *Use units of* 10^2 *gigawatts for WP* – see the entry for 2016 in the table below.

Answer: The model would be given by the function

$$\text{WP} = 301 \cdot (1.332)^t$$

where WP is in gigawatts and t is in years after 2016.

C. Compute the values of WP predicted by your model for the years $2017-2022$. Round to 2 decimal places. About how many years will it take for WP to reach approximately double the 2016 level?
Answer: The projected values are

Year	Wind Power (in gigawatts)
2017	400.93
2018	534.04
2019	711.34
2020	947.51
2021	1262.08
2022	1681.09

You can think of these as the values at the start of each year. The amount of electricity generated by wind power will reach twice the 2016 level when

$$602 = 301 \cdot (1.332)^t, \quad \text{so} \quad t = \frac{\log(2)}{\log(1.332)} \doteq 2.4 \text{ years}$$

(about 4/10 of the way through the year 2018).

D. How many years will it take for wind power generation to reach 2.0×10^3 gigawatts according to your model?
Answer: We solve

$$2000 = 301 \cdot (1.332)^t$$

and find

$$t \doteq \frac{\log(6.6445)}{\log(1.332)} \doteq 6.6 \text{ years}$$

(that is, about 6/10 of the way through the year 2022).