MONT 104N – Modeling the Environment
Chapter 1 Project
September 10, 2018

Introduction

This project is inspired by a similar project developed for the book *Quantitative Reasoning and the Environment* by Greg Langkamp and Joseph Hull (Prentice Hall, 2007), but uses different and more recent data.

The Greenland ice sheet is estimated to contain about 1/9 of the total freshwater ice on the planet. Many recent observations indicate that melting is well under way at present. The goal of this first chapter project is for you to use information from the map in Figure 1 to make an estimate of the total amount of ice contained in the Greenland ice sheet, and to understand where the predictions of the effects of total melting are coming from.

Because this ice is presently on land, if it should all melt and flow into the oceans, sea levels around the world would rise. This is different from the situation for the Arctic sea ice, or much of the ice in the shelves surrounding Antarctica, which rest on sea water. Melting in those cases would not appreciably change sea levels because that ice is floating on and supported by water already. This is a consequence of Archimedes' Principle.[1] According to Archimedes, *a floating object displaces a volume of the fluid that has the same mass as the object.* So if a chunk of ice floating in a body of water melts, the water level will not change. This can be verified by an easy experiment in a pot or bathtub. This is good since there are ongoing changes in the Antarctic ice shelves. For instance, on July 12, 2017 the Larsen C ice shelf, with an area of about $44,200$ square kilometers, broke off from the Antarctic coast and became what is probably the largest iceberg ever observed by humans. There are important freshwater ice sheets covering the land mass of Antarctica as well as these sea ice formations, and those make up most of the remaining fresh water ice on the Earth.

As we will see, there is a large volume of water contained in the Greenland ice sheet. For this reason, its fate is of more than passing interest for all humans. Many of our major cities and settled coastal areas occur in regions that are close enough to the current sea level that any significant increase would cause major disruptions. For instance, the *highest* (natural) elevation of any point on the island of Manhattan in New York City is only 81 meters ($\doteq$ 265 feet) above sea level and most of the island lies much lower than that. The effects of the relatively small increase in average sea level that have already occurred were evident, for instance, in the flooding of lower Manhattan that happened during "super-storm Sandy" in October, 2012.

Trying to forecast how the ice sheet will evolve over time can be intricate because weather conditions producing melting change from year to year, while the ultimate fate of the ice sheet depends on long-term trends such as the levels of carbon dioxide and other "greenhouse gases" in the atmosphere, deposition of soot onto the ice, and other factors.[2] In every year, some of the

[1] Stated in Proposition 5 of his work *On Floating Bodies*

[2] Since the first two of these are connected with human burning of fossil fuels for energy, there is also a political

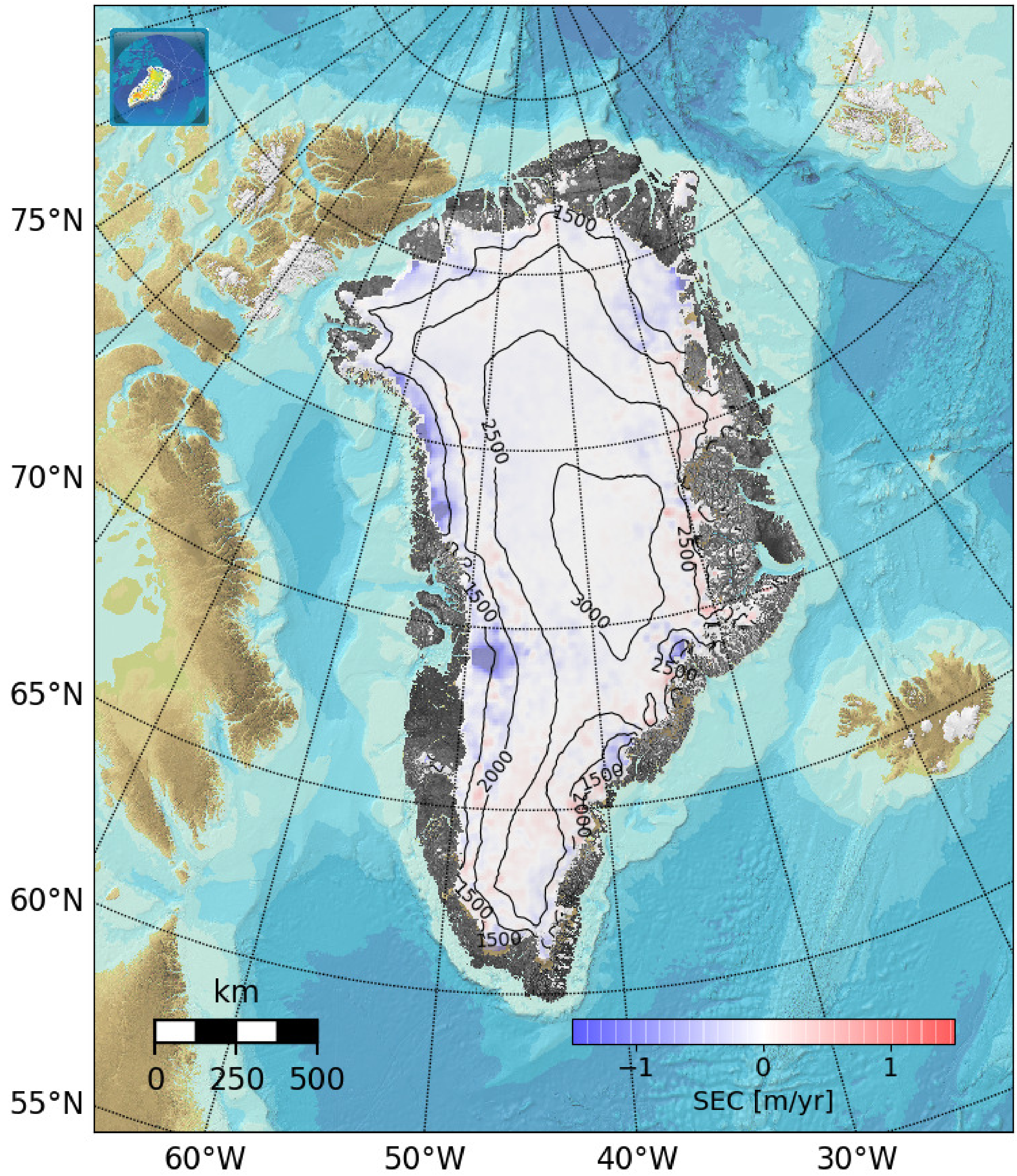

Figure 1: Contour Plot of Surface Elevation of the Greenland Ice Sheet, With Estimated Changes Over the Period 2013-2017 Derived From Cryosat-2 Satellite Measurements, source: European Space Agency CCI Ice Sheets Greenland project.

ice sheet flows into the ocean and produces (or "calves") icebergs. At the same time, new snow falls onto the ice sheet and creates new ice. While these two processes were apparently balancing each other out over a long period of time until recently, they have not been balancing in many recent years. For instance, the summer of 2012 was much warmer than the long-term average up to that time in Greenland and it produced large and well-publicized melting events for which the ice was not replaced by new snowfall. On the other hand, the years 2016 and 2017 were cooler and included larger than average snowfall amounts in parts of Greenland. These were large enough, in fact, that there might even have been a small net increase in the volume of the ice sheet those years. However, this was not evenly distributed geographically and melting also did take place those years. The National Snow and Ice Data Center (a research institute funded by NASA, the National Oceanographic and Atmospheric Administration (NOAA) and the National Science Foundation) maintains a web site that provides daily updates on key measurements related to the Greenland ice sheet and information about historical trends. The data for the 2018 season shows that there were much larger than usual surface melting areas in late May and early June, but the rest of the year so far (I am writing this on July 18) has been close to the 1980-2010 median in surface melting area.

The Data

Refer to the map of Greenland showing the surface elevation of the ice covering most of its land area in Figure 1. We will make use of the *surface elevation contour lines* to make our estimate. These are the black lines superimposed on the map of Greenland labeled with the numbers 1500, 2000, 2500, 3000. Each of these numbers represents a height above sea level in meters. The area included inside one of these contour lines all has surface elevation *at least* the number associated with that contour.

Our methods will yield rough estimates or approximations of the ice volume. They will be based on the following simplifying assumptions:

- Greenland is not exactly flat. Some of the areas at the highest elevation are mountains in the region on the eastern coast that are not covered by the ice sheet. The bedrock under the ice sheet is not exactly flat either. But in fact the ice sheet is so massive that it has forced large sections of interior bedrock *below* sea level, while other sections are at or above sea level. Hence, to do our estimate, we will average things out and make a simplifying assumption that the ice sheet is resting on a perfectly flat base at sea level.

- As another simplifying assumption, we will treat the ice sheet as though it is one gigantic block of ice with no cracks, no hollow spaces, etc.

- Even though it appears as a large area on the familiar Mercator projection maps you have probably seen, that map projection distorts areas of regions near the poles and makes them look much larger than they actually are. Greenland does not make up a very large a portion of the surface area of the roughly spherical Earth. As a result we will not lose too much if

component in some public discussions of the issues here.

we simply estimate areas as though they corresponded to areas on the flat map (i.e. without trying to take the curvature of the Earth into account). The distance scale marked in the legend of the map can be used, together with a ruler, to approximate linear dimensions of regions.

- For the purposes of this estimate, let's assume that the surface elevation inside each contour line up to the next higher elevation value is *equal* to the number printed over the contour in each case. Also assume all of the white areas outside the 1500 meter contour line have ice at surface elevation 1000. The dark colored areas outside that region also have some ice, but we will neglect them.
- The violet and pink colors refer to the right hand scale at the bottom. They show an estimated *surface elevation change* in m/yr over the period 2013 to 2017, as estimated by measurements from the Cryosat-2 satellite launched by the European Space Agency.

Questions

The chapter project will involve investigating the following questions and writing up your results as directed.

(A) Estimate the ice areas with surface elevation 1000 m, 1500 m, 2000 m, 2500 m, and 3000 m. Explain how you are doing this in a clearly-written paragraph. Note: There are *many ways* to do this in a reasonable fashion and there is not just one right answer! Suggestion: print out a paper copy of the page with the map and overlay a transparency marked with 250 km by 250 km squares. Count the equivalent number of such squares in each region to estimate the area. You don't need to get super-detailed or picky, but be as accurate as possible.

(B) Multiply each of your area estimates by the depth estimate to get a volume estimate. (Don't forget that we are assuming the ice sheet starts at an elevation of 0 m.) Add the ice volume estimates to get a total volume and express in units of *cubic kilometers*.

(C) As a "reality check" for your method, the total volume of the Greenland ice sheet is often estimated to be about $3,000,000$ cubic kilometers. How close did you come to that? Is your method systematically underestimating or overestimating the ice volume? If you cannot necessarily say either way because different aspects of what you did would tend to pull the estimate in opposite directions, explain.

(D) When ice melts, the volume of the water that is produced is slightly smaller:

$$\text{(volume of water)} \doteq (.92) \cdot \text{(volume of ice)}$$

(see Exercise 17 in the text). Imagine that an amount of water equal to the melted form of your estimate of the volume of the ice is added to the oceans all at once. How much would sea levels rise as a result? One way to estimate that is to use the same idea as in part (3) of Example 1.5 in the text. What is the total surface area of the oceans on the Earth? (You should look this up online. If you find different estimates, how will you choose which one to use? Explain your thinking.) If the water from the melted Greenland ice sheet was spread evenly over that area, how deep would it be, in meters? Would the actual change in sea level be less than or greater than this estimated height? Explain.[3]

(E) Find an elevation contour map of Manhattan. Use that information to estimate what portions of that island would be under water if all the freshwater ice sheets melted.

(F) (Preview of Chapter 2.) Using the violet and pink shaded regions, estimate whether the total volume of the ice sheet increased or decreased over the period 2013 to 2017. Note that you will need to use both the estimated areas of regions, and the colors representing the the surface elevation changes in m/yr to do this. Suggestion: if an area of increase balances out an area of decrease, approximately, you can ignore both of them for your estimate.

Assignment

Write up your group's solutions for these questions as a project report (just one writeup per group). Include all of your calculations, the version of the map you used to estimate the areas of the different regions of the ice sheet, and your answers to all the "explain" portions of the questions above.

[3] Technical note: In case you are worried about the fact that we are ignoring the spherical shape of the Earth, this method is actually sufficient (i.e. accurate enough) for our purposes because of the fact that the depth of the water would be much smaller than the radius of the Earth. Here's one way to think about it: If there is no land area, then adding the water from the melted ice sheet is equivalent in mathematical terms to changing the radius of the spherical Earth from r to $r + \Delta r$, with Δr representing the (unknown) depth of the new water, much smaller than r itself. The volume of the added water is the difference between the volume of the larger sphere of radius $r + \Delta r$ and the volume of the original sphere of radius r:

$$\frac{4\pi(r+\Delta r)^3}{3} - \frac{4\pi r^3}{3} = 4\pi r^2 \times \Delta r + 4\pi r \times (\Delta r)^2 + \frac{4\pi(\Delta r)^3}{3}.$$

Since we assume Δr is much smaller than r, the last two terms on the right are negligible in size compared to the first term and we obtain an estimate

$$\text{volume of added water} \doteq 4\pi r^2 \times \Delta r = \text{surface area of sphere} \times \Delta r.$$

The change in sea level is then approximated by

$$\Delta r \doteq \frac{\text{volume of added water}}{\text{surface area of sphere}}.$$

The same idea works even if the water covers only a portion of the surface area of the sphere. The denominator would be replaced by the surface area of the portion of the Earth covered by water in that case.