MONT 104N – Modeling the Environment
Answers for additional practice problems for final exam
December 11, 2018

Caution: These answers were "banged out" at speed, so I cannot really guarantee that they are 100% accurate. Ask me if anything looks wrong.

Chapter 1.

5.
  (a) $44,200 \text{ km}^2 = 44,200 \text{ km}^2 \cdot (.621)^2 \text{ mi}^2/\text{km}^2 \doteq 17045 \text{ mi}^2$.
  (b) The thickness in kilometers is .350 km, so the volume in cubic kilometers $= 44,200 \cdot .350 \doteq 15470\text{km}^3$. Then converting to cubic feet, 1 cubic kilometer is about $(5280/1.61)^3 \doteq (3280)^3$ cubic feet, so 15470 cubic kilometers is $\doteq 3.53 \times 10^{10}$ cubic feet.

11.
  (a) $\log_{10}(5.34689) \doteq .7281$, $\log_{10}(53.4689) \doteq 1.7281$ and $\log_{10}(534.689) \doteq 2.7281$. What is happening is that when you multiply by a power 10^k, then $\log_{10}(10^k) = k$ is *added* to the value of the logarithm, by part (1) of Proposition 1.8.
  (b) $\log_7(34.333) = \log_{10}(34.333)/\log_{10}(7) \doteq 1.8172$ (see formula (1.2) on page 12.)
  (c) $\ln(100.3) \doteq 4.6082$. (This could also be computed as $\log_{10}(100.3)/\log_{10}(e)$, where e is the base of the natural logarithms.)

13.
  (a) $20 \cdot \log_{10}(5000/20) \doteq 47.96$ dB.
  (b) If $L = 1$ dB, then $1 = 20 \cdot \log_{10}(p_m/20)$, so $p_m = 20 \cdot 10^{1/20} \doteq 22.44$ micropascals. Similarly if $L = 2$ dB, then $p_m \doteq 25.18$ micropascals, and if $L = 10$ dB, then $p_m \doteq 63.25$ micropascals.
  (c) About $20 \cdot 10^{150/20} \doteq 6.32 \times 10^8$ micropascals. This shows a logarithmic scale in action!

Chapter 2.

2.
  (a) First student: absolute error $= 3.44 - 3.40 = .04$ meter. Percent relative error $= \frac{3.44-3.40}{3.40} \times 100\% \doteq 1.2\%$. Second student: absolute error $= .44 - .40 = .04$ meter. Percent relative error $= \frac{.44-.40}{.40} \times 100\% = 10\%$.
  (b) The first student was more accurate because the percent relative error was smaller.
  (c) The absolute error tells you how far off the measurement is from the true value. Absolute errors are important one measurement at a time. The percentage relative error expresses the error as a percent of the exact value. So, percentage relative errors are more useful for comparing accuracies of different measurements.

(d) Precision of a set of measurements is not the same as accuracy. Accuracy measures how far the measurements are from an exact value. Precision measures how close the measurements are to each other.

7.

(a) Boston: 13321.3 people per square mile $\doteq$ 34530.1 people per square kilometer. Chicago: 11868.1 people per square mile $\doteq$ 30763.3 people per square kilometer. Miami: 11198.7 people per square mile $\doteq$ 29028.1 people per square kilometer. NYC: 27016.3 people per square mile $\doteq$ 70029 people per square kilometer. Philadelphia: 11233.6 people per square mile $\doteq$ 29118.7 people per square kilometer. San Francisco: 17246.4 people per square mile $\doteq$ 44704.4 people per square kilometer.

(b) In 2016, from page 26, we see that the population of New York was estimated at 8.54×10^6. That gives a population density of 28222.1 people per square mile. The percent change in population density from 2010 (reference) to 2016 (comparison) was

$$\frac{28222.1 - 27016.3}{27016.3} \times 100\% \doteq 4.5\%$$

The numbers would come out the same if the densities per square kilometer were used since the conversion factors from square miles to square kilometers in the numerator and denominator would cancel.

Chapter 4.

6.

(a) $y - 5.2 = 7 \cdot (x - 3.2)$ or $y = 7x - 17.2$.

(b) $y - 1 = -1/7(x - 0)$ or $y = -x/7 + 1$.

8. $g(x)$ is the linear one because that is the only function for which the slopes between pairs of points are always the same $= -2.25$. The equation is $y - 1.525 = -2.25(x - 1.1)$ or $y = -2.25x + 4$.

9. Kudzu area $= \frac{7.4-0}{2018-1876}(t - 1876)$ (in units of 10^6 acres, t in years).

Chapter 5.

2. All of these are solved by taking logarithms:

(a) $x = \frac{1}{2}\frac{\log_{10}(28.3/4.5)}{\log_{10}(3.4)} \doteq .7513$.

(b) $x = \frac{\log_{10}(3.5)}{\log_{10}(4) - \log_{10}(2)} \doteq 1.8074$.

(c) $x = \frac{\log_{10}((7.9-5.6)/2.8)}{\log_{10}(7.4)} \doteq -.09828$.

4.

(a) $f(t) = 8.54 \cdot (1.15)^t$.

(b) $f(t) = 3.5711 \cdot (1.03)^t$.

6.

(a) doubling time $= \frac{\log_[10](10/5)}{\log_{10}(1.25)} \doteq 3.106$

(b) doubling time $= \frac{\log_[10](36.6/18.3)}{\log_{10}(3.4)} \doteq .5664.$

7. As in the special cases in 6 above, we solve for t in the equation

$$2 \cdot Q(0) = Q(0) \cdot a^t$$

to find the doubling time, t_2. Dividing both side by $Q(0)$, then taking logarithms, we get

$$t_2 = \frac{\log_{10}(2)}{\log_{10}(a)}.$$

8. This follows from rules for exponents. From the previous problem, $\log_{10}(a) = \frac{\log_{10}(2)}{t_2}$, so

$$a = 10^{\log_{10}(a)} = 10^{\log_{10}(2)\cdot\frac{1}{t_2}} = 2^{\frac{1}{t_2}}.$$

Then raising both sides to the t power and multiplying by $Q(0)$ we get

$$Q(t) = Q(0) \cdot a^t = Q(0) \cdot \left(2^{\frac{1}{t_2}}\right)^t = Q(0) \cdot 2^{\frac{t}{t_2}}.$$

9.

(a) Proceeding as in Example 5.8 in the text, from the half-life of this isotope:

$$Q(28.8) = \frac{1}{2} \cdot Q(0) = Q(0) \cdot a^{28.8}$$

so $\log_{10}(a) = \frac{\log_{10}(.5)}{28.8} \doteq -.0105$ and $a = 10^{-.0105} \doteq .9762$. Then the model is

$$Q(t) = Q(0) \cdot (.9762)^t.$$

(b) We solve for t:

$$.01 \cdot Q(0) = Q(0) \cdot (.9762)^t,$$

so $t = \frac{\log_{10}(.01)}{\log_{10}(.9762)} \doteq 191.2$ years.

Chapter 7.

1. The percentage change gives $Q(n+1) - Q(n)Q(n) \times 100 = r$, so $Q(n+1) = \left(1 + \frac{r}{100}\right) \cdot Q(n)$. Hence if we start from $t = 0$, we get

$$\begin{aligned}
Q(1) &= \left(1 + \frac{r}{100}\right) Q(0) \\
Q(2) &= \left(1 + \frac{r}{100}\right) Q(1) = \left(1 + \frac{r}{100}\right)^2 Q(0) \\
Q(3) &= \left(1 + \frac{r}{100}\right) Q(2) = \left(1 + \frac{r}{100}\right)^3 Q(0)
\end{aligned}$$

and so forth. The general pattern is

$$Q(n) = \left(1 + \frac{r}{100}\right)^n Q(0).$$

Note that this is an exponential function of n, with $a = \left(1 + \frac{r}{100}\right)$.

4.
(a) $Q(n) = 3.4 \cdot (1.8)^n$ by problem 1 above.
(b) Here we want to use the general solution for affine first order equations from (7.4) on page 129:

$$Q(n) = \left(4.3 + \frac{(-.03)}{(.78 - 1)}\right)(.78)^n - \frac{(-.03)}{(.78 - 1)} = 4.4364 \cdot (.78)^n - .1364.$$

8.
(a) From the equation, $r = .03$ and $r/M = .006$, so $M = 5$. Since $Q(0) = .8 < M$, the solution will increase in a sort of S-shape and tend toward M as a horizontal asymptote as n increases.
(b) $r = .34$ and $r/M = .0009$, so $M = 377.8$. Since $Q(0) = 420 > M$, the solution will decrease toward M.
(c) $r = .86$ and $r/M = .0048$, so $M = 179.2$. Since $Q(0) < M$, this is similar to part (a).

Chapter 8.

3. The equations are

$$\begin{aligned} A(n+1) &= .4A(n) + .25B(n) + .5C(n) \\ B(n+1) &= .4B(n) + .4A(n) \\ C(n+1) &= .33C(n) + .2B(n) \end{aligned}$$

7.
b. Say the fraction of the I group that recovers is $\gamma_1 I(n)$ and the fraction that dies is $\gamma_2 I(n)$ for some constants γ_1, γ_2. The R compartment is correspondingly split between R_1 and R_2. The equations would be

$$\begin{aligned} S(n+1) &= S(n) - \beta S(n)I(n) \\ I(n+1) &= I(n) + \beta S(n)I(n) - \gamma_1 I(n) - \gamma_2 I(n) \\ R_1(n+1) &= R_1(n) + \gamma_1 I(n) \\ R_2(n+1) &= R_2(n) + \gamma_2 I(n) \end{aligned}$$