MONT 104N – Modeling the Environment
Information on Final Exam
December 7, 2018

General Information and Groundrules

As announced in the course syllabus, the final exam for our Montserrat seminar will be given at the announced time for MWF 9:00am classes:

Thursday, December 13, 8:00am to 10:30am

The exam will be held in our regular class room.

- This will be an individual exam. No sharing of information in any form will be permitted during the exam.
- You may bring (and should use) a calculator during the exam, but no other electronic devices. *Know how to compute logarithm and exponential values* using your calculator in addition to basic arithmetic.
- If you are well-prepared and work steadily, I expect the exam will take 1 to 1 1/2 hours (60 to 90 minutes) to complete. However, you will have the full 2 1/2 hour period (150 minutes) to work if you need that much time.
- This will be a *comprehensive* exam. The exam questions may cover concepts and techniques from any section of the course (Chapters 1,2,3,4,5,6,7,8 in the Modeling and Data Analysis textbook). But I will only ask about topics we have actively discussed in class, the problem sets, or the projects.
- There will be four or five mathematical problems (each possibly with a few separate parts). These questions will be similar to things you have seen on the problem sets or the group projects. Some sample exam questions are given later in this document. I will post solutions for those in a few days – my recommendation is to try working out solutions first on your own, then use the solutions to check your work.
- The exam will also include an essay question (worth about 1/3 of the total points) on the set topic below. It should take you about 20 or 30 minutes to produce a good, detailed answer for that question. So it will be necessary to spend a sufficient portion of your preparation time on deciding what you want to say.
- I will be available during the reading period at the following times for "last minute" questions:
  - Monday, December 10, 2:00pm - 5:00pm
  - Tuesday, December 11, 8:00am - 11:50am
  - Wednesday, December 12, 8:00am - 10:00am and 1:00pm - 3:00pm

Essay Topic

In general terms, what is a mathematical model? Describe in general terms what they are, how they are constructed, and how they are used. Give examples of three different types of mathematical models we have studied this semester. Even if mathematical models don't capture *every feature* of a real world situation, why is it still important to develop them and understand the information we get from them? For instance, what conclusions about use of natural resources did we derive by looking at logistic models with various types of harvesting in the Chapter Project from Chapter 7? As another example, how are mathematical models important in understanding our choices of which energy sources to use? Why is it important to understand how radioactive substances decay? What type(s) of model(s) that we discussed would apply to describe that process? What are some of the issues involved with using radioactive decay to generate electricity–that is, why is this not a "no-brainer" as a solution to the problem of CO_2 buildup in the atmosphere from fossil fuel burning?

Sample Mathematical Questions

Note: These were essentially the mathematical questions from the final exam I gave in MONT 100N in Fall 2017. The questions this year will be different, but comparable in topics covered, in length, in level of difficulty, etc.

I. In 1990, forests covered 4.047×10^9 hectares of the Earth's surface. By 2000, forest area had decreased to 4.038×10^9 hectares. Assuming that the decrease in forest area is *linear*, and that it will continue at the same rate into the future, in this problem you will develop a linear model for the forest area remaining as a function of t = years since 1990.

A. Determine the slope for the linear model of the forest area. What are the units of the slope?

B. What is the linear equation modeling the forest area as a function of t = years since 1990.

C. According to your model, in what year will the forest area reach 4.0×10^9 hectares?

D. According to the United Nations Food and Agriculture Organization, the actual forest area remaining in 2010 was 4.033×10^9 hectares. How close is the prediction your model from part B gives for the forest area in 2010?

II. Wind power has emerged as a fast growing source of energy for electrical power generation in recent years. In 2016, the generating power of wind turbines installed around the world was about 301 gigawatts and it was increasing at about 33.2% per year.

A. The typical English unit of power is the horsepower. 1 horsepower = 7.457×10^{-7} gigawatts. Convert 301 gigawatts to the equivalent number of horsepower.

B. Construct an exponential model for WP = wind power generation as a function of t = years after 2016. *Use units of* 10^2 *gigawatts for WP* – see the entry for 2016 in the table below.

C. Fill in the table of values for WP below with values predicted by your model for the years 2017 – 2022. Round to 2 decimal places. About how many years will it take for WP to reach approximately double the 2016 level?

$Year$	2016	2017	2018	2019	2020	2021
WP	3.01					

D. How many years will it take for wind power generation to reach 20×10^2 gigawatts according to your model?

III. Suppose that a population of fast-reproducing insects in an area has a natural growth rate of 5% per month from births and deaths, and that there is a net migration *loss* of 20 individuals per month.

A. Write a difference equation that models this situation.

B. Using an initial value $P(0) = 500$, determine the populations in months $1, 2, 3, 4, 5$ according to the model you stated in part A and record the values in the following table (round any decimal values to the nearest whole number)

n	0	1	2	3	4	5
$P(n)$	500					

C. What happens to the population in the long run as n increase? Does it tend to a definite value?

IV. Answer the following questions with a few sentences each.

A. If you are fitting an exponential model to a data set (x_i, y_i) "by hand," you start by transforming the data to what new form (X_i, Y_i)?. If the best fit regression line for the transformed data is $Y = mX + b$, what is the corresponding exponential model? (You may assume the logarithms have base 10 as we discussed in class.)

B. If you are fitting an power function model to a data set (x_i, y_i) "by hand," you start by transforming the data to what new form (X_i, Y_i)? If the best fit regression line for the transformed data is $Y = mX + b$, what is the corresponding power function model? (You may assume any logarithms have used base 10 as we discussed in class.)

C. What does the R^2 statistic in measure in linear regression? How did we use it? Explain what it would mean, for instance if $R^2 = 1$.

D. What type of chart (scatterplot, pie chart, bar chart, etc.) would be most useful to describe the composition of a forest if there 5 different types of trees present in different concentrations per acre? Explain, and illustrate your answer with a chart if a typical acre of forest contains 10 oaks, 12 maples, 5 pines, 2 hemlocks, and 1 chestnut.

E. What difference equation would model a population undergoing logistic growth if the population was growing at about 4% per year when the population is much smaller than the carrying capacity $M = 400$ of the habitat?

F. Describe the SIR model for infectious disease outbreaks and give the corresponding difference equations. What is an *epidemic* and given the constants β, γ and $S(0)$, what determines when an epidemic will occur?

G. What feature of the solutions of the Lotka-Volterra equations is considered a confirmation that this model is capturing an important aspect of real-world predator-prey interactions?

Full solutions for all of these will be posted at the start of exam week so you can check your work. Some additional practice problems to look at are:

1. Chapter 1/5, 11, 13 (Note: the conversion factors you need are given in the text. I will supply any information like that you would need for questions like these ones.)
2. Chapter 2/2, 7
3. Chapter 4/6, 8, 9
4. Chapter 5/2, 4, 6, 7, 8, 9
5. Chapter 7/1, 4 (know the general solution for affine first-order difference equations and how to determine equilibrium solutions algebraically; I will not give you that formula), 8 – sketch the solutions by hand in rough, qualitative terms
6. Chapter 8/3, 7 (only parts (a) and (b) would be suitable for an exam question, though!)

Note: I could also ask you "qualitative" questions relative to the process of fitting a linear, exponential, or power function model to data like one of the questions from the midterm, including questions about the process of transforming the data via logarithms for the exponential and power law cases. (See IV A and B above for what I mean by this.) I *can't really* ask you to compute a regression equation for fitting a linear or exponential model to data, of course, since apart from one simple linear example, we only did those computations with the spreadsheet.) I could give you the equation of a regression line, though, and ask you to give a model prediction for an x-value not occurring in the data.