A First Mathematical Modeling Problem

A small house is to be built on flat ground in the shape of a rectangular solid with exterior walls that are 9 meters high. The region where the house will be built has extremely severe winters, during which cold winds blow primarily from due north (think Minnesota or North Dakota in the U.S. or Norway or Sweden in Europe). One wall of the house will face directly north (it has "northern exposure") and bear the full force of the cold winds. The south-facing wall will be sheltered from the winds, while the other two, east- and west-facing walls will be somewhat less sheltered.

Annual heating costs for the house satisfy the following:

- Each square meter of exterior wall with the northern exposure adds \$50 Dollars to the annual heating cost.
- Each square meter of exterior wall with an eastern or western exposure adds \$25 to the annual heating cost.
- Each square meter of exterior wall with a southern exposure adds \$12 to the annual heating cost.

(A) Denote by L the length of the north- and south-facing walls. (Note these are equal because the house is a rectangle.) If the length of the east- and west-facing walls are constant (for instance at 20 meters), what can you say about the total annual heating cost as a function of L.

(B) Now suppose that the length of the north- and south-facing walls is constant (for instance at 30 meters) and W represents the length of each of the east- and west-facing walls. What can you say about the total annual heating cost as a function of W.

(C) If L and W are both free to vary, what is the total annual heating cost as function of L and W together?

(D) Now assume the total area of the "footprint" of the house is to be exactly 200 square meters. Express the total annual cost of heating the house in terms of L (the length of the north- and south-facing walls). This is a mathematical model(!)

(E) Give examples of two different real-world questions that could be addressed using this model.