

MONT106Q – Mathematical Thinking  
The “Monty Hall Problem”  
September 9, 2016

*Background*

The following situation figures in several different ways in *the curious incident of the dog in the night-time*.

On the TV game show *Let’s Make a Deal*, the old host Monty Hall would frequently play the following game with contestants:

- The contestant would be shown three large closed doors and told that behind one was a *new car*, while *live goats* were behind the other two.
- The contestant would select one of the doors.
- At this point Monty (who always *knew* where the car and the goats were) would reveal one of the goats, behind one of the doors the contestant *had not picked* – note that with two goats, there was always at least one such door to reveal.
- He then offered the contestant the option of *staying with the original door they had picked* or *switching to the other unopened door*.
- When they made their choice, what was behind the door they finally selected was revealed and the game concluded.

(In case you’re wondering, when the contestant “won” a goat, I don’t think the show ever discussed what happened next. I suppose it’s possible some contestants might have really wanted a goat instead of a car; others may have been at least willing to settle for a goat. But in most cases, the show probably allowed the contestant to go home without the goat even if they had “won” it(!))

In 1990, in her *Parade* magazine column, Marilyn Vos Savant responded to a question from a reader and explained that the best strategy for the contestant was to *always switch doors when offered the choice*. Immediately hundreds of people, including a bunch of well-known professional mathematicians wrote to her saying she had made a mistake. They claimed there shouldn’t be any difference between switching or keeping the original choice because at that point the chances were 50/50 the person had the door with the car.

To start today, we want to do a simulation to see who may have been right about this.

*Directions*

- Work in groups of two or three (8 groups of two, assuming everyone is present).

- In each group, one person will be Monty Hall, the other will be a contestant, and the third will be a recorder of the results.
- Your group will have three playing cards to represent the three doors (two two's – the goats, and an ace – the car).
- **Round 1:** Repeat 20 times and record the results – how many times the contestant won the car and how many times the contestant won a goat: “Monty” shuffles the cards, deals them out *face down*, and notes which cards are in which locations. The “contestant” makes the initial choice but leaves the card face down, Monty reveals a “goat” (not the card chosen by the “contestant”), the contestant *keeps the original choice* and reveals it.
- **Round 2:** Repeat 20 times and record the results – how many times the contestant won the car and how many times the contestant won a goat: “Monty” shuffles the cards, deals them out *face down*, and notes which cards are in which locations. The “contestant” makes the initial choice but leaves the card face down, Monty reveals a “goat” (not the card chosen by the “contestant”), the contestant *switches to the other unrevealed card* and sees the result.
- When you have completed 20 iterations in Round 1, then 20 iterations in Round 2, we'll combine results from all of the groups and try to understand what happened.
- If “Monty” forgets which cards are the goats and reveals the wrong card, or if the rules above are not followed in some way, discard the results of that iteration and *try again*.
- If you want to switch roles between the two rounds, go ahead. “Monty” needs to pay more careful attention to what's going on so that a correct card is revealed in each repetition(!)