MONT 106Q – Mathematical Thinking
Problem Set 2 – Egyptian Mathematics
due: no later than 5:00pm, September 26, 2016

I. *Egyptian multplication and division.*

A) Multiply by successive doubling (that is, "the Egyptian way"): 434×127. (Make it easier on yourself by a wise choice of the multiplier and multiplicand!)

B) Multiply by successive doubling: 252×59.

C) Divide by the Egyptian method: $167 \div 18$.

D) What *advantage(s)* does the Egyptian method of multiplication have over the method commonly used now? What shortcomings?

II. *Egyptian fractions.* A part of the table of conversions of $2/n$ into unit fractions from the Rhind papyrus is given in the handout from class on Wednesday, September 21. Each row gives an expansion

$$\frac{2}{n} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}$$

for one of the odd integers $3 \le n \le 101$. (if there is no entry for c and/or d, then those terms are not present).

A) Verify that the rows for $n = 13$ and $n = 29$ are correct.

B) Use the table to find a decomposition of $\frac{4}{27}$ into unit fractions.

C) Show that for any positive integer n,

$$\frac{2}{n} = \frac{1}{n} + \frac{1}{2n} + \frac{1}{3n} + \frac{1}{6n}.$$

Yet this expansion appears only once in the table from the Rhind papyrus. For which n did the compiler of the table fall back on this form? Can you see why they might have preferred to avoid using this if possible?

III. *The* **seked** *of a pyramid.* The Egyptians measured the steepness of a face of a pyramid by the ratio of the "run" (measured in palms) to the "rise" (measured in cubits). There were 7 palms in a cubit. This ratio was called the *seked* of the pyramid, measured in units of *palms per cubit.* (See page 110 in *The Crest of the Peacock.*)

A) Problem 56 of the Rhind papyrus asks the solver to find the *seked* of a pyramid 250 royal cubits tall, with a square base 360 royal cubits on a side. The answer is given as $5 + \frac{1}{25}$ hands per royal cubit. The computation is given by Joseph, but not really explained. Explain what is going on there with diagrams. In particular, why does the calculation start out by taking half of the 360?

B) The great pyramid at Giza (built by the Pharaoh Khufu, finished about 2560 B.C.E., and the only one surviving of the "seven wonders of the ancient world") has a base 440 cubits on a side and a height of 280 cubits. What is its *seked*?

IV. *An Egyptian approximation to π.* We take π to be the ratio between the area of a circle and the square of the radius: $\pi = \frac{A}{r^2}$.

A) In the Rhind papyrus, the area of a circle with diameter d is given as

$$A_{circ} = \left(\frac{8}{9}d\right)^2.$$

(This is not exactly correct, but yields decent approximate results.) What is the implied approximation to the value of π here?

B) Here's one way this may have been derived. Take a square of side 9 units, divide the edges into three equal pieces, and cut off four corner triangles with side length 3 to make an octagon. The octagon is pretty close in area to the circle inscribed in the original square. Explain why the octagon has area $63 \doteq 8^2$, and then explain how the approximation $A_{circ} = \left(\frac{8}{9}d\right)^2$ would result.