MONT 106Q – Mathematical Thinking
Problem Set 1 – Ancient Mayan mathematics from the *Dresden Codex*
Due: September 19, 2016, no later than 5:00pm

Background

The ancient Mayans (see Joseph, *The Crest of the Peacock*, pp. 66 and following) used a positional *"mixed" vigesimal*, or (mostly) base-20, number system. We will see why the "mixed" is needed here in question A below. They wrote their base-20 "digits" $0, 1, 2, \ldots, 19$ in an extremely economical way using just three different symbols – a round dot for 1, a long bar for 5, and a "shell-shaped" symbol for 0.

In this problem set you will look at a page from one of the surviving ancient Mayan books. There are four or five of these; we'll be looking at one called the *Dresden Codex*[1]. A Mayan codex is a folding book made from tree bark, with writing created by brush and ink on a surface prepared with a kind of plaster. What you will do in this assignment is to decipher and try to interpret a portion of the contents.

See the .pdf file of three pages accompanying this problem set on the course homepage. There is also a color photo of the specific page we'll look at in its correct orientation posted there – as you will see the original uses both red and black colors and some of the glyphs are more legible than in the black and white photocopy. We are going to concentrate on the material on the *lower half* of the first (*left-hand*) page in the black and white version. Take a good look at that page to start, and note that a great proportion of what is there consists of numbers(!) How should we read them and what do they mean, though?

Questions

A) Let's start with the numbers below the two rows of other glyphs. Each is a three-digit mixed base-20 number, with the digits stacked *vertically*, reading up the column. It is known that the ancient Maya scribes used *a slightly odd system* for recording *numbers of days in calendar calculations*. In the notation for numbers of days, instead of a pure base 20 system, a modified or *mixed* system was used in which the digits were numbers of 1's, 20's, $18 \times 20 = 360$'s, 18×20^2's, etc. (This system is possibly explained by the fact that $18 \times 20 = 360$ is closer to the number of days in a solar year than $20^2 = 400$

[1] The Dresden Codex was created some time after the end of the Classic Maya period (around 900 CE), likely copying an earlier source. It was one of the few Mayan books to escape being burned by the Spanish during their colonization of Central America (when they tried to exterminate the indigenous culture). It is kept in a museum in the German city of Dresden, hence the name. (A fascinating sidelight is that the damage across the top of the pages is not ancient. It is water damage incurred in the course of extinguishing a fire in that German museum. The fire occurred during fire-bombing of Dresden by the Allies in WWII in February 1945. The museum workers cared enough about preserving this artifact to save it even though most of the city of Dresden was destroyed and perhaps 25,000 civilians lost their lives in these attacks.)

is.) Calculate the numbers in these six columns using this "modified" base-20 system. (*Hint:* The number in the second column from the left should work out to 5551.)

B) One way to explore a series of numbers is to combine or work with them arithmetically in various ways. A very useful thing to do with a list of numbers in a lot of situations is to find *the differences* between successive pairs in the list. What do you get if you do that here moving from left to right across the six columns?

C) Look at the numbers in the *last two rows at the bottom of the page* and decipher them. How do they relate to what you saw in part B? What could these mean? Well, the Mayanists who deciphered the Dresden Codex noticed something interesting. The *lunar month* (the time from one new moon to the next new moon) is approximately 29.5 days. How are the number 177 that appears among the differences and 29.5 related?

D) The glyphs on the rows *in between* the rows of numbers from the previous parts are names of particular days in the Sacred Round or *tzol'kin* portion of the Mayan calendrical system discussed starting on page 68 in Joseph (also see the course homepage for a graphical representation of the *tzol'kin*). Here's how they translate top to bottom within each column:

Column 1: 4 Ik, 5 Akbal, 6 Kan.

Column 2: 12 Cauac, 13 Ahau, 1 Imix.

Column 3: 7 Cib, 8 Caban, 9 Etznab

Column 4: 2 Ben, 3 Ix, 4 Men.

Column 5: 10 Oc, 11 Chuen, 12 Eb.

Column 6: 2 Etznab, 3 Cauac, 4 Ahau.

What is the pattern in each column? How are the two dates on the "ends" related to the one in the middle?

E) *What does the table mean?* The page we looked at is part of a section of the table that is thought to deal with a particular astronomical phenomenon that has interested people of many different cultures. Each of the "middle" dates in the columns from part D is correlated with a date on which it is known that a *lunar eclipse* (in most cases partial, not total) occurred in the years 770 - 772 C.E. that would have been visible in the Mayan lands.

Some astronomical background: Lunar eclipses occur at a full moon when the earth is between the sun and the moon and appears to block out the moon's light. If the moon's orbit was not inclined slightly to the plane of the earth's orbit about the sun (called the *ecliptic* in astronomy) then eclipses would occur every month (and as a result, they would probably have been much less alarming to pre-scientific societies).

But because of the inclination of the moon's orbit, eclipses can only happen when the moon is crossing the ecliptic, and this happens twice in each lunar month, at the "ascending node" and "descending node." Now, if the moon is not "lined up" with the sun and the earth at that time, there is no eclipse. However, if the moon is near the ascending (resp. descending) node at the time of an eclipse, it will be near the decending (resp. ascending) node about half a year later. There is *a chance that an eclipse will occur again* at the nearest new or full moon. This will usually be six lunar months, or approximately 177 days, after the first eclipse (this can be closer to 178 days in some cases). Several eclipses may follow at 177- (or 178-) day intervals. The lunar month is not exactly 29.5 days long, though, so eventually the differences build up enough and another eclipse in the series cannot occur. At the time of the last eclipse in a series, though, the closest new moon to the node may arrive after five lunar months, approximately 148 days. (It is important to realize that the Earth is rotating about its axis while all this is going on, so the eclipses in a series like this might not all be visible from any one particular location on the Earth.)

Based on this astronomical information, in 1913, an astronomer named *Martin Meinshausen* came up with an interpretation of what this table in the Dresden codex probably was for the Mayans. By thinking about this information and your transcriptions of the numbers from the codex page carefully, you should be able to see what his interpretation was. Write a paragraph describing *Meinhausen's interpretation of what this part of the Dresden Codex is about.* Start with a clearly stated topic sentence, then explain how the calculations you did in the previous parts and the given information about eclipses support your claim.