MONT 106Q – Mathematical Thinking
Information for Final Exam
December 7, 2016

General Information

The final exam for this course will be given in during the period 8:00am - 10:30am on Thursday, December 15. The final will be similar in format to the midterm exam but perhaps 1.5 times as long. I expect that if you are well prepared and you work steadily, then you should be able to finish the exam in about 75 minutes. However, you will have the full 150 minute period to work on the exam if you need that much time.

The exam will cover all of the mathematical material we have studied since the start of the semester as well as the CHQ common readings *Othello* and the *Flatland*. For the mathematical questions, you may use a calculator (a graphing calculator is fine; graphing functions will not be used, though). No use of cell phones, I-pods, I-pads, or any other electronic devices beyond a calculator will be allowed during the exam. *If your only calculator is on your phone and you want to use one, then you will need to borrow a calculator for this exam.*

Format

Approximately 60% of the exam will consist of several short mathematical problems similar in format and content to the ones you have seen on the problem sets, together with a few multiple choice questions and/or short answer questions on the mathematical topics. The remaining 40% will be a short essay concentrating on aspects of the two CHQ common readings above. The mathematical topics are, in particular:

1. General ideas about number systems – positional systems (like ours) vs. non-positional systems (like Roman numerals, for instance), the *base* of a positional system, etc. Know how to interpret base-10 and base-2 numbers and give the base-10 form of a number expressed in base-2 and vice versa. Understand the distinction between numbers, numerals (i.e. the symbols used to represent numbers) and number-words in a language.

2. Egyptian mathematics:

    a. Know the approximate historical period represented by our primary Egyptian mathematical sources (the Rhind and Moscow mathematical papyri).
    b. Know the basic idea of their number system and the number symbols for 1's, 10's, 100's, 1000's (see p. 85 in *The Crest of the Peacock*)
    c. "Egyptian" multiplication and division by repeated doubling (you will not need to use the Egyptian symbols for this – compute with modern Hindu-Arabic numerals, but using the Egyptian methods).
    d. Egyptian computations with unit fractions via tables of representations like the one from the Rhind papyrus on page 95 of *The Crest of the Peacock* or the handout

we discussed in class); also know that there are many different representations for fractions as sums of unit fractions.

e. Be prepared for problems like the ones on Problem Set 2.

3. The Classic Mayan number and calendar systems (see Problem Set/Discussion 1) and the Lunar Eclipse Table from the Dresden Codex: For questions about computations within the *tzol'kin* calendar, as on the midterm exam, I would give you the list of "month" names. There is no need to memorize them.
   a. Know how to work with the "mixed" base-20 system the Classic Maya used.
   b. Understand the *tzol'kin* calendar with 13 "months" of 20 days each and how days are identified in that system.
   c. Understand what the information on the page from the Dresden Codex we deciphered is and how it shows the mathematical thinking of the Classic Maya.
4. The Warlpiri kinship system: The structure of the 8 sections in the society, which sections can marry, how the section membership of children is determined, the mother and the father relations.
5. Eulerian circuits in graphs: know how to determine when a graph has an Eulerian circuit.
6. Symmetric strip patterns: know the different types of symmetry transformations such a pattern can have and how to recognize when they are present. Also know how to identify the type of the pattern in the standard classification, *given the "decision tree" that is posted on the course homepage*. If I ask you about this (and I probably will!), I will supply a version of that diagram in the test pages.
7. Groups: Know the mathematical definition of a *group*, and be able to discuss two different types of examples where that structure appeared in examples of mathematical thinking we discussed this semester.
8. The rules and play of the game in *mancala*. Know especially how to determine whether a given position is one where you can force the end of the game by playing all your stones into your store.

Essay

The prompt for the essay question will be *one of the following*. (**NOTE:** I will choose the question for the exam, so you will probably want to work out responses to both of these prompts to be well-prepared.) Aim for about 2 hand-written pages for your answer. In grading essay questions on an in-class exam, I will be looking just at the content of what you say. I might mark technical errors (misspellings, etc.) but those will not be taken into account in the grading.

1) What are some of the *boundaries/borders* that Shakespeare uses to structure his plot and characters in *Othello*? For instance, what is a "Moor"? What does the word mean and how does the fact that Othello is a Moor create a boundary between him and Venetian society? Where does the action of the play take place and how does that include a political or physical border? Who besides Othello has transcended

borders and how? Does transcending those borders end well or result in tragedy? What conclusions can we draw here?

2) A key episode in *Flatland* involves the interaction between the Sphere from Spaceland and the Square from Flatland where the border of the two-dimensional Flatland is literally transcended. What does the Sphere do to demonstrate the existence of a third dimension? Why does the Sphere say, "Listen, no stranger must witness what you have witnessed?" What is the Square's reaction to being taken into space? Find three words in the text that have the prefix *omni-* and explain what they mean and how they relate to this episode. Why does the Sphere say that omnividence doesn't make a person a God? What qualities does he say make one more divine? What is the Square's reaction?

Some Practice Mathematical Questions

I.

A) Express the base-10 number 231 in base-2.

B) (Note: we did not discuss this, but you should be able to see how it works if you think carefully about how we add in our base-10 number system.) How do you add in base-2? Describe it in words first. Then, for instance, what is

$$101011101 + 100000111$$

(Express the sum first in base-2, then check your work by converting everything to base-10 and adding that way.)

II. Compute "the Egyptian way"

A) 67×103

B) $83 \div 14$ (For this one, don't worry about trying to follow a systematic procedure when you get to the fractional part, just get a sum of a whole number and a legal sum of unit fractions any way you can!) You can refer to a $2/n$ table like the one from the Rhind papyrus on page 95 of *The Crest of the Peacock* or the handout we discussed in class. I would give you a copy of that table for a question of this sort.

III.

A) Why is the Maya number system called a mixed-base system? Explain.

B) How would a Maya scribe represent the number 561 (write it in left-to-right format)?

IV.

A) Suppose a Warlpiri woman from section 1 marries a man from section 5. To what section do their children belong?

B) What is the *mother relation* m as a permutation of the 8 sections? (That is, given the section of the child, to which section did the mother belong)?

C) Same as B, but for the father relation f.

D) Is mf the same as fm in the Warlpiri kinship system? (That is, is the mother of one's father (mf) in the same section as the father of one's mother (fm)?)

E) What is the name of the mathematical group that describes the "algebra" of the m and f relations for the Warlpiri?

V. Let K_n be the graph with n vertices and one edge joining every pair of distinct vertices (this is called the *complete* graph on n vertices in graph theory).

A) Draw K_2, K_3, K_4, K_5 and K_6. (The simplest forms of these will place the vertices equally spaced

B) Which of the graphs from your part A have Eulerian paths?

C) Now in general, for which n does K_n have an Eulerian path? (Your answer should say exactly which n's give a graph with that property.)

VI. Suppose you are playing *mancala* and it is your turn. Your 6 pits contain numbers of stones left to right as follows: 0,5,3,0,0,0. Is it possible to get all 8 stones to your store before the other player gets to play? If so, show how to do it, step by step. If not, explain why not.

VII. For each of the following symmetric strip patterns:

A) Say in words all the different kinds of symmetries are present (i.e. does it have translation, glide reflection, vertical axis reflection, or horizontal axis reflection symmetry?).

B) Consult the "decision tree" from the course homepage and say what is the type of the symmetry group.