MONT 106Q – Mathematical Thinking
Solutions for Final Exam Review Problems
December 7, 2016

Some Practice Mathematical Questions

I.

A) Express the base-10 number 231 in base-2.
Answer: 11100111.

B) (Note: we did not discuss this, but you should be able to see how it works if you think carefully about how we add in our base-10 number system.) How do you add in base-2? Describe it in words first. Then, for instance, what is

$$101011101 + 100000111$$

(Express the sum first in base-2, then check your work by converting everything to base-10 and adding that way.)
Solution: The idea is that you align the numbers on the right and add digit by digit just as in base-10. If the corresponding base-2 digits in the numbers you are adding are $0, 0$, then that digit in the sum is 0; if the corresponding base-2 digits are $1, 0$ or $0, 1$, then that digit in the sum is 1. The main difference is if the corresponding base-2 digits in the two numbers are $1, 1$, then the sum has a 0 in that digit and you carry a 1 into the next digit. Just as in base-10 addition, you think of any digits that get carried as added to the digit in the first number, and then you follow the rules as above. This means, for instance, that if the two numbers both have a 1 in some digit, and you are carrying a 1 into that digit, then you get a 1 in that digit in the sum, *and* you carry a 1 into the next digit. For instance:

$$101011101 + 100000111 = 101100100$$

Check: 101011101 is $1+4+8+16+64+256 = 349$ and 100000111 is $1+2+4+256 = 263$, so in base-10, $349 + 263 = 612$. Now $612 = 512 + 64 + 32 + 4$.

II. Compute "the Egyptian way"

A) 67×103
Solution:

$$(*)1 \times 103 = 103$$
$$(*)2 \times 103 = 206$$
$$4 \times 103 = 412$$
$$8 \times 103 = 824$$
$$16 \times 103 = 1648$$
$$32 \times 103 = 3296$$
$$(*)64 \times 103 = 6592$$

Then using the starred entries, $67 \times 103 = 103 + 206 + 6592 = 6901$.

B) $83 \div 14$ (For this one, don't worry about trying to follow a systematic procedure when you get to the fractional part; just get a sum of a whole number and a legal sum of unit fractions any way you can!) You can refer to a $2/n$ table like the one from the Rhind papyrus on page 95 of *The Crest of the Peacock* or the handout we discussed in class. I would give you a copy of that table for a question of this sort.
Solution: We compute with 14 to get 83:

$$1 \times 14 = 14$$
$$2 \times 14 = 28$$
$$4 \times 14 = 56$$

We can stop there since doubling again gives $112 > 83$. Now $83 - 56 = 27$, so $\frac{83}{14} = 5 + \frac{13}{14}$. There are any number of ways to proceed from here, yielding different sums of distinct unit fractions that add up to $\frac{13}{14}$. One way:

$$\frac{13}{14} = \frac{12}{14} + \frac{1}{14} = \frac{6}{7} + \frac{1}{14}$$

Now we can think $\frac{6}{7} = \frac{2}{7} + \frac{2}{7} + \frac{2}{7}$. From the $2/n$ table, this gives

$$\frac{1}{4} + \frac{1}{28} + \frac{1}{4} + \frac{1}{28} + \frac{1}{4} + \frac{1}{28}$$

Adding two of the $\frac{1}{4}$'s gives $\frac{1}{2}$. Adding two of the $\frac{1}{28}$'s gives $\frac{1}{14}$. That $\frac{1}{14}$ plus the $\frac{1}{14}$ from the $\frac{13}{14}$ gives $\frac{1}{7}$. So the final answer (this way) is

$$5 + \frac{1}{2} + \frac{1}{4} + \frac{1}{7} + \frac{1}{28}$$

This satisfies the Egyptian requirements because it's a sum of distinct unit fractions.

III.

A) Why is the Maya number system called a mixed-base system? Explain.
Solution: It's mixed-base because it represents numbers using $1, 20, 18 \times 20, 18 \times 20^2, \ldots$. It's mainly base 20, but with the "wrinkle" of the 18 starting in the third digit.

B) How would a Maya scribe represent the number 561 (write it in left-to-right format)?
Solution: $561 = 1 \times 360 + 10 \times 20 + 1$, so the number would be represented left-to-right like this:

IV.

A) Suppose a Warlpiri woman from section 1 marries a man from section 5. To what section do their children belong?

Answer: See the diagrams on p. 71 of *Ethnomathematics.* The child's section is determined by the to from the mother's section, so the child is in section 4.

B) What is the *mother relation* m as a permutation of the 8 sections? (That is, given the section of the child, to which section did the mother belong)?
Answer: The child's section determines the mother's section like this:

$$\begin{gathered} \text{child} \to \text{mother} \\ 1 \to 3 \\ 2 \to 4 \\ 3 \to 2 \\ 4 \to 1 \\ 5 \to 8 \\ 6 \to 7 \\ 7 \to 5 \\ 8 \to 6 \end{gathered}$$

C) Same as B, but for the father relation f.
Answer: The father relation is:

$$\begin{gathered} \text{child} \to \text{father} \\ 1 \to 7 \\ 2 \to 8 \\ 3 \to 6 \\ 4 \to 5 \\ 5 \to 4 \\ 6 \to 3 \\ 7 \to 1 \\ 8 \to 2 \end{gathered}$$

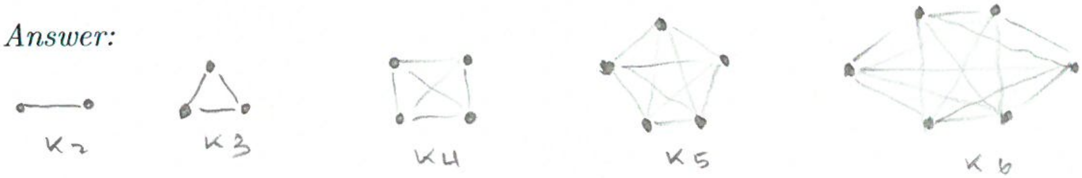
D) Is mf the same as fm in the Warlpiri kinship system? (That is, is the mother of one's father (mf) in the same section as the father of one's mother (fm)?)
Answer: No they are not the same. For instance (from the answers in parts B and C, if you are in section 1, your mother was in section 3 and her father was in section 6. But your father was in section 7 and his mother was in section 5. That one difference is enough to say $mf \neq fm$.

E) What is the name of the mathematical group that describes the "algebra" of the m and f relations for the Warlpiri?
Answer: This is called *dihedral group of order 8.*

V. Let K_n be the graph with n vertices and one edge joining every pair of distinct vertices (this is called the *complete* graph on n vertices in graph theory).

A) Draw K_2, K_3, K_4, K_5 and K_6. (The simplest forms of these will place the vertices equally spaced

Answer:



B) Which of the graphs from your part A have Eulerian paths?

Answer: K_2 does if you start at one vertex and end at the other, K_3 does since all vertices have degree 2 which is even, K_4 does not since all the vertices have odd degree and there are 4 of them, K_5 does, and K_6 does not.

C) Now in general, for which n does K_n have an Eulerian path? (Your answer should say exactly which n's give a graph with that property.)

Answer: The ones that do are K_2, and K_n for all n odd. The reason is the that when n is odd, from each vertex there are edges to all of the *other* $n-1$ vertices in the graph, and if n is odd, then $n-1$ is even.

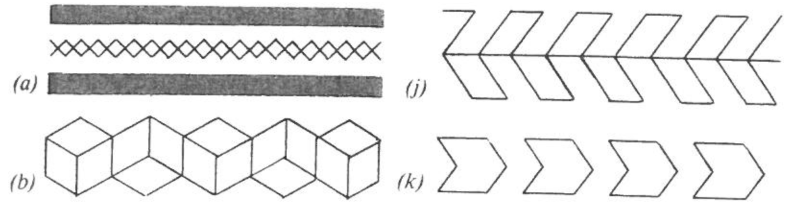
VI. Suppose you are playing *mancala* and it is your turn. Your 6 pits contain numbers of stones left to right as follows: 0,5,3,0,0,0. Is it possible to get all 8 stones to your store before the other player gets to play? If so, show how to do it, step by step. If not, explain why not.

Answer: No it is not possible. Playing from the rightmost "harvestable" pit each time (underlined), this is what happens:

$$0, \underline{5}, 3, 0, 0, 0$$
$$0, 0, 4, 1, 1, \underline{1}$$
$$0, 0, \underline{4}, 1, 1, 0$$
$$0, 0, 0, 2, 2, \underline{1}$$
$$0, 0, 0, 2, \underline{2}, 0$$
$$0, 0, 0, 2, 0, \underline{1}$$
$$0, 0, 0, 2, 0, 0$$

and you are "stuck."

VII. For each of the following symmetric strip patterns:



A) Say in words all the different kinds of symmetries are present (i.e. does it have translation, glide reflection, vertical axis reflection, or horizontal axis reflection symmetry?).
Answer: Pattern (a) has all of them. Pattern (b) has translation and glide reflection symmetry, plus vertical axis reflection symmetry. Pattern (j) has only translation and glide reflection symmetry. Pattern (k) has translation, glide reflection, and horizontal axis reflection symmetry.

B) Consult the "decision tree" from the course homepage and say what is the type of the symmetry group.
Answer: (a) is $pmm2$, (b) is $pma2$, (j) is $p1a1$, (k) is $p1m1$