MONT 106Q – Mathematical Thinking
Discussion on Graphs and Eulerian Paths
October 28, 2016

Background

The Bushoong people of Zaire make sand drawings of several kinds illustrating the results on Eulerian paths in graphs that we discussed last time. Today, we will consider two types extending those on pages 33 to 37 of Ascher's book.

Questions

I. The Bushoong sand figure in Figure 2.2a on page 35 is just one of a whole family of figures constructed according to a similar pattern. We will use the following notation for these. We will call the design in Figure 2.2a the figure B_2 because the vertices form *two* diamonds side by side. There is a similar figure B_k for each integer $k \geq 1$. For instance, here are the figures B_1 on the left and B_3 on the right:

Note: B_1 has 1 central diamond; B_3 has 3 of the central diamonds.

A) How many vertices and edges do each of the figures B_1, B_2, B_3 have?
B) What about the general case? How do the numbers of vertices and edges depend on k? Can you prove your result? (Hint: the technique of *mathematical induction* is the quickest way if you have seen that in a course before; if not ask me and I'll tell you the idea.)
C) Explain why *all* of the B_k figures have Eulerian paths.

II. On pages 36 and 37, Ascher discusses the way Bushoong children actually trace out the figures shown in Figure 2.2b and c. These types have Eulerian paths starting at one vertex of odd degree and ending at the other vertex of odd degree. Ascher gives what amount to *algorithms* for drawing these figures, expressed as series of drawing commands.

A) Exactly what is an *algorithm*? Look up a precise definition and explain how it relates to what Ascher is saying here.
B) Draw the figures for $N = 6$, $N = 8$, and $N = 12$ using the general algorithm at the bottom of page 36. You will find it easiest to use graph paper for this – rotate the paper by 45 degrees so that the grid of the graph paper can be used to draw the lines in the ul, ur, dl, and dr directions as directed.
C) Now look at the modified algorithm at the top of page 37. How does that generalize to a general even number N (the given special case is $N = 10$).
D) Following your algorithm from C, draw the figures specified by $N = 6$ and $N = 8$.