Montserrat 105Q– Mathematical Journeys: From the Unknown to the Known
Syllabus – Spring 2016

Professor: John Little
Office: Swords Hall 331
Office Phone: (508) 793-2274
Office Hours: M 2-3pm, T 10am-12noon, W 2-4pm, R 2-4pm, F 11am-12noon
and by appointment
Course Homepage: `http://mathcs.holycross.edu/~little/Mont201516/Spring.html`

2015-2016 Core Human Questions Cluster theme

How, then, shall we live when the journey may be as important as the origin or the destination?

General description

This course is a part of the Core Human Questions cluster of the Montserrat program. This means that at various times we will be addressing one or several of the following:

- The general aims of Montserrat – continued development of your thinking, writing, and communication skills, and the connection of living, learning and doing in college education,
- The common readings and theme of this year's Core Human Questions cluster.
- The specific subject matter of this spring's seminar – gaining an appreciation of the intellectual journeys that people take within mathematics and the ways mathematicians create knowledge and communicate their understanding to others via statistical reasoning.

Subjects for this spring's seminar

First, we will study Charles Darwin's record of one of the most important journeys in the history of science – his 5-year voyage of exploration in the 1830's as a civilian traveling with the crew of the British warship *H.M.S. Beagle*. Darwin's purpose on this trip was to make measurements of longitude, and observations of the flora, fauna, and geology of various places in South America, Pacific Islands, etc. gather samples, and describe them in writing as the ship circumnavigated the globe. What he saw on the way influenced his thinking and formed the nucleus of the revolutionary ideas on evolution by natural selection that he published in his later work *The Origin of Species*.

Analogous journeys of discovery were also underway within mathematics at roughly the same time. For instance, the "undiscovered country[1]" of non-Euclidean geometries was brought to light through the work of Janos Bolyai, Nikolai Lobachevsky, and Carl Friedrich Gauss. This development had the consequence that the propositions Euclidean geometry could no longer reasonably be taken as self-evidently truths about the physical universe. This called into question just what

[1]See *Hamlet*, Act 3, Scene 1, line 87. It's amazing how many common phrases stem from the text of this play!

Euclid was doing. Somewhat later, motivated by disturbing questions about the theory behind differential and integral calculus, mathematicians attempted to lay new groundworks for mathematics in the theory of sets. Eventually, paradoxical results emerged that called this whole enterprise into question and threw the subject into a *crisis of foundations*.

By the early 20th century, this train of thought led to a realization that the strict axiomatic, deductive method exemplified by Euclid's *Elements* has definite limitations. Specifically, the *Incompleteness Theorem* proved by Kurt Gödel says, essentially, that within any axiomatic system powerful enough to encompass the arithmetic of the natural numbers, there will always be *true* statements that *cannot be proved within the system*. We will begin this semester by briefly looking at some of this history, through a wonderful and unique *graphic novel* called *Logicomix* by Apostolos Doxiadis and Christos Papadimitriou. We will read this coupled with some additional sections of Davis and Hersh's *The Mathematical Experience* in which the issues involved are described in a bit more technical detail.

Considering the level of controversy it produced and the time and effort expended by many mathematicians and logicians trying to resolve it, it is perhaps surprising that the ultimate outcome of the crisis in foundations has been *inconclusive*, to say the least! None of the solutions proposed to give mathematics provably correct foundations has been successful! As a result, we do not even know whether the mathematics we have is *consistent*, and even worse, that sort of consistency is essentially *undecidable* within mathematics itself (i.e. there is no way to prove whether or not our mathematics will eventually produce contradictory results!) Yet pure mathematicians have gone on proving new theorems all the same. Applied mathematicians use those tools more than ever and they appear to "work" extremely well as descriptions of all sorts of real world phenomena.

Where does this leave us? A recent (and, to me, very convincing) point of view is that the most productive future developments of mathematics will lie, not in the arid abstractions and illusory certainty of Euclidean-style axiomatic mathematics, but rather in deeper understanding of how *chance and randomness* influence real-world processes. (This is the message of a rather controversial article called *The Dawning of the Age of Stochasticity* by David Mumford that we will read a couple of weeks into the semester.) At the same time, this opens up the possibility of answering more realistic questions of definite interest in everyday life: How do political pollsters conduct surveys concerning the upcoming Presidential election and how reliable are the results they report? Does a eating a high-fat diet increase our risk of developing cancer? How likely is Joe DiMaggio's 56-game hitting streak to be surpassed in our lifetimes? Finding patterns and answering such questions can be seen as journeys from the unknown to the known. We will use the book *Naked Statistics* by Charles Wheelan (supplemented by some more technical lecture notes I will prepare and distribute) to study this side of mathematics. We will take a detailed and occasionally critical look at the ways reasoning about probabilities is used to identify patterns via statistics. In the process, you will develop an appreciation of the power and the limitations of statistical thinking and learn to analyze claims backed by statistics.

We will take short breaks from our study of probability and statistics to discuss the other two CHQ common texts, the film *The Curious Case of Benjamin Button*, and Shakespeare's incomparable masterpiece *Hamlet* and we will get to see the Theater Department's production of *Hamlet* this spring in conjunction with our reading of the play.

Some groundrules

Most class meetings will be devoted to discussions, oral presentations, or work in smaller groups, so your active participation will be important for the success of what we do.

- Unless specifically directed otherwise, please *turn off* all cell phones, tablet or laptop computers and other similar electronic devices for the duration of each class meeting – your full attention and participation will be necessary.
- No distracting or "provocative" clothing, headgear, or other personal items in class, please.
- In on-campus events, off-campus trips, etc. you are representing this seminar, the Montserrat program, and, by extension, Holy Cross in a wider community. Take responsibility and regulate your behavior accordingly.
- In discussions, there may be times that you disagree with Prof. Little or with a classmate. Please feel free to express that disagreement and be prepared to say why you disagree and back up your ideas with evidence. But please keep the conversation civil and respectful.

Course objectives

The major objectives of the seminar will be:

1. To acquaint you with some of the history of the "crisis in foundations" in mathematics in the late 19th and early 20th centuries.
2. To develop some of the basic ideas of probability and the techniques of statistics that use those ideas to make inferences about processes involving chance and randomness.
3. *Individually*, to further your development as speakers and writers.
4. *As a group*, to create an atmosphere where civil and constructive conversations can take place. Many of the topics we discuss will have controversial aspects and reasonable people can have very different viewpoints on them. Recognizing that, everyone (including Prof. Little, of course) should listen carefully and seek to understand where others are coming from, especially when your first inclination might be to disagree. Good academic *writing* can also be seen as a conversation between the writer and others who have thought about similar questions. So this way of doing things will carry over into the way we approach writing assignments as well.
5. To participate in and contribute to the common activities of the Core Human Questions Cluster of the Montserrat program.

Texts

The texts for the course are (CHQ common texts first)

1) Darwin, Charles, *The Voyage of the Beagle*, abridged and edited by Millicent Selsam (distributed as .pdf file on course homepage), out of print.

2) *The Curious Case of Benjamin Button*, film to be shown on Thursday, February 18 at 7:00pm in Seelos Theater.
3) Shakespeare, William, *Hamlet*, Folger Shakespeare Library updated edition, Simon and Schuster, 2012
4) Davis, P. and Hersh, R. *The Mathematical Experience* Study Edition, Birkhauser, 2012.
5) Doxiadis, A. and Papadimitriou, C. *Logicomix*, Bloomsbury Press, 2009.
6) Wheelan, C, *Naked Statistics*, W.W. Norton, 2014.

There will also be a few shorter readings and lecture notes posted on the course homepage as .pdf files for downloading. See course schedule for more information about when will be looking at each one.

Course schedule

A detailed day-by-day course schedule and listing of required and suggested outside events will be maintained on the course homepage (and will be accessible through the Moodle course management system). That listing is a tentative, evolving schedule, so it may change and you will probably want to refer to it frequently. Any important changes will also be announced in class well in advance.

Assignments and grading

1) *Midterm exam* (20 % of course grade) – tentative date: Friday, April 1.
2) *Final project paper and oral presentation* (30 % of course grade) – replaces final exam. More details about this to come.
3) *Problem sets* (about 6 assignments – 20 % of course grade)
4) *Two roughly 3-5 page papers*, and other shorter writing assignments. (Information and guidelines to be distributed later) (20 % of course grade)
5) Class participation (10 % of course grade)

I will be keeping your course average in numerical form throughout the semester, and only converting to a letter for the final course grade. The course grade will be assigned according to the following conversion table (also see Note below):

- A – 94 and above
- A- – 90 - 93
- B+ – 87 - 89
- B – 84 - 86
- B- – 80 - 83
- C+ – 77 - 79
- C – 74 - 76
- C- – 70 - 73

- D+ – 67 – 69
- D – 60 - 66
- F – 59 and below.

Note: Depending on how the class as a whole is doing, some downward adjustments of the above letter grade boundaries may be made. No upward adjustments will be made, however. (This means, for instance, that an 85 course average would never convert to a letter grade of B- or below, although it might be a B+ in some circumstances.) If you ever have a question about the grading policy or your standing in the course, don't hesitate to ask me.

Advice on how to succeed in this class

A good "work ethic" is key. As you should be able to tell from the course description above, you do not need to be a "math genius" to do well in this course. But you will need to put in a consistent effort and keep up with the reading and assignments.

Come to class. Unless you are deathly ill, have a genuine family emergency, are away at a game or meet of a college athletic team, etc. plan on showing up here at 9:00 am every Monday, Wednesday, and Friday this semester. Many of the class meetings will be structured around discussions or student presentations. Your participation is expected and needed for the success of the course!

Take notes and use them. This may seem obvious, but it is worth saying! Used intelligently, your notes can be a valuable resource as you work on problem sets and prepare for the exams.

Use the texts and class notes actively. Reading about mathematics is not like reading a novel. You will probably need to read and think over things more than once. You may want to work through examples to understand some of the topics that we do.

Set up a regular study schedule and work at a steady pace. It's not easy to play catch-up in a mathematics course (even when the course is part of a first-year program with additional goals beyond the mathematics). You should expect to budget at least 6 hours in a typical week for work outside of class. The best way to use your time is to do a few problems, some reading from the books, and reviewing of class notes every day.

Most importantly, if you are having difficulty learning something, get help as soon as possible. You can do this by asking questions during class (any time something isn't clear), or seeing me during office hours.

Statement on academic integrity

All education is a cooperative enterprise between teachers and students. This cooperation works well only when there is trust and mutual respect between everyone involved. To be become an engaged and advanced learner, you must be able to think and work both independently and in concert with your peers. The College academic honesty policy states: "As an institution devoted

to teaching, learning, and intellectual inquiry, Holy Cross expects all members of the College community to abide by the highest standards of academic integrity. Any violation of academic honesty undermines the student-teacher relationship, thereby wounding the whole community. The principal violations of academic honesty are plagiarism, cheating, and collusion.

Plagiarism is the act of taking the words, ideas, data, illustrative material, or someone else's statements, and presenting them as one's own without full and proper acknowledgment.

Cheating is the use of improper means or subterfuge to gain credit or advantage. Forms of cheating include the use, attempted use, or improper possession of unauthorized aids in any examination or other academic exercise submitted for evaluation; the fabrication or falsification of data; misrepresentation of academic or extracurricular credentials; and deceitful performance on placement examinations. It is also cheating to submit the same work for credit in more than one course, except as authorized in advance by the course instructors.

Collusion is assisting or attempting to assist another student in an act of academic dishonesty. The full statement on Academic Honesty in the College Catalog is available at

`http://www.holycross.edu/catalog/acad_program.pdf`

If you do not know how to correctly cite reference materials, consult with your professor, the campus Writers Workshop, or visit one of the links below:
Holy Cross Department of History:

`http://academics.holycross.edu/history/academichonesty.htm`

Wadsworth Cengage, "Plagiarism Prevention Zone"

`http://college.cengage.com/english/plagiarism_prevention.html`

NOTE: If in doubt about what you plan to do or write violates academic honesty, PLEASE ASK! The temptation to engage in an act of academic dishonesty may arise, but the chance possibly to enhance a single grade is not worth the loss of your personal integrity.

Specific guidelines for this course

In this course, all examinations will be closed-book. No consultation of outside sources of information or sharing of information with other students in any form will be permitted during exams. In group discussions, close collaboration with the other members of your group is expected. For the papers, please refer to the specific guidelines for each assignment regarding what is acceptable and how to cite sources other than the course texts if that is allowed. In general, some discussion of ideas for your paper with classmates and with Prof. Little will be acceptable as you prepare to write. But your final written work should be your own and any ideas you use from other sources should be acknowledged. On individual problem sets, discussion of the questions with other students in the class and with me during office hours is allowed, *even encouraged*. However, your final problem solutions should be prepared individually and the wording and organization of your final problem solutions should be entirely your own work. Moreover, if you do take advantage of any of

the above options for discussion of problems with others, you will be required to state that fact in a footnote accompanying the problem solution. Failure to follow these rules will be treated as a violation of the College's Academic Integrity policy.