Montserrat 104Q– Mathematical Journeys: From the Known to the Unknown
Syllabus – Fall 2015

Professor: John Little
Office: Swords Hall 331
Office Phone: (508) 793-2274
Office Hours: M 1-2pm, T 10am-12noon, W 1-3pm, R 2-4pm, F 11am-12noon,
and by appointment
Course Homepage: `http://mathcs.holycross.edu/~little/Mont201516/Fall.html`

2015 Core Human Questions Cluster theme

How, then, shall we live when the journey may be as important as the origin or the destination?

General description

This course is a part of the Core Human Questions cluster of the Montserrat program. This means that at various times we will be addressing one or several of the following:

- The general aims of Montserrat – continued development of your thinking, writing, and communication skills, and the connection of living, learning and doing in college education,
- The common readings and theme of this year's Core Human Questions cluster.
- The specific subject matter of this seminar – gaining an appreciation of the intellectual journeys that people take within mathematics and the ways mathematicians create knowledge and communicate their understanding to others via proofs.

Subjects for this fall's seminar

The first full week of the semester will be devoted to discussing the first cluster common reading – the *Odyssey* of Homer – the story of one hero's long and adventure-filled journey home from the Trojan War. First written down around 800 BCE as a record of a long tradition of oral performance of poetry on this theme, the *Odyssey* has earned a place as the prototype of the epic journey in Western literature.

After this, we will turn to the mathematical portion of the course. Does it surprise you to learn that people are constantly adding to what we know about mathematics? How does that happen, and how do new ideas come to be accepted? Ever since the time of the ancient Greeks (but significantly later in history than the time of Homer), *proofs* of mathematical results have played a key role in this. To relate this to the theme of our Montserrat cluster, the process of finding and writing down a proof can (very profitably) be seen as a sort of *intellectual journey*. The origin of this journey is what is known at the outset, the destination is the result to be proved. The journey itself is reflected in the sequence of logical steps that get us from the known to the (previously) unknown. Mathematicians would say these journeys into the unknown are often just

as important as the destinations, mostly because the knowledge gained in finding a proof can also lead to other unexpected results and new understanding.

But how does this relate to the more practical aspects of mathematics that you have studied before? For some mathematicians, in fact, the process of gaining understanding and finding proofs comes to be more important and interesting than any possible practical application of those ideas. Our starting point in this part of the course will be the testimony of one particular British mathematician of the 20th century, G. H. Hardy, who left a very personal testament of what mathematics meant to him in his book *A Mathematician's Apology*. Hardy's "take" on the practical side of mathematics remains controversial, to say the least. But it should provide us with an opportunity to discuss the ethical implications of doing any scientific or mathematical work.

Next, to get a feeling for the powerful attraction this sort mathematical work can have on some (maybe even some of you!), we will take a close look at Book I of Euclid's *Elements*. (This work is also the source of several particular proofs that Hardy discusses, but his examples come from later sections.) Much as the *Odyssey* has served as a prototype for stories about journeys, the *Elements* has served as a prototype for textbooks of mathematics, right down to the present day. As we will see, Book I is a very tightly-organized sequence of theorems and proofs leading to a masterful proof of a familiar result from plane geometry that all of you have seen (although you may not know a proof for it).

Following the October vacation, we will take a short break from our mathematical journey for the second cluster common reading, *Huckleberry Finn* by Mark Twain. Twain's novel has been seen as the source of most of distinctively American literature and we will see how Huck and Jim's trip down the Mississippi River in pre-Civil War America is another sort of epic journey.

The second mathematical portion of the course will be devoted mainly to the question: How do we *find proofs*? We will have seen by this point in the course that this step in the process is not addressed at all in Euclid's work. Many, many generations of students have found this frustrating, to say the least. And indeed, having only the ultimate destination or *finished product* in the form of a polished proof, *à la Euclid*, is arguably not the most productive way to learn to make use of mathematical ideas, although teachers of mathematics have different opinions about what might replace it(!) In this part of the course, we will look at the "messier" cycle of conjectures (informed guessing), attempted proofs, counterexamples, and refutations by which actual proofs come into being through the interaction of groups of real people.

Our final cluster common reading, Cheryl Strayed's memoir *Wild* will round out this first semester.

No mathematical background beyond high school geometry and algebra is required for this course, but students should be open to new points of view and willing to think deeply.

Some groundrules

Most class meetings will be devoted to discussions, oral presentations, or work in smaller groups, so your active participation will be important for the success of what we do.

- Unless specifically directed otherwise, please *turn off* all cell phones, tablet or laptop computers and other similar electronic devices for the duration of each class meeting – your full

attention and participation will be necessary.

- No distracting or "provocative" clothing, headgear, or other personal items in class, please.
- In on-campus events, off-campus trips, etc. you are representing this seminar, the Montserrat program, and, by extension, Holy Cross in a wider community. Take responsibility and regulate your behavior accordingly.
- In discussions, there may be times that you disagree with Prof. Little or with a classmate. Please feel free to express that disagreement and be prepared to say why you disagree and back up your ideas with evidence. But please keep the conversation civil and respectful.

Course objectives

The major objectives of the seminar will be:

1. To acquaint you with the role of proofs in mathematics, to learn some famous and influential examples of proofs from Euclid's *Elements*.
2. To acquaint you with some of the process by which proofs are developed by mathematicians and to give you the experience of finding proofs of your own.
3. *Individually*, to further your development as speakers and writers.
4. *As a group*, to create an atmosphere where civil and constructive conversations can take place. Many of the topics we discuss will have controversial aspects and reasonable people can have very different viewpoints on them. Recognizing that, everyone (including Prof. Little, of course) should listen carefully and seek to understand where others are coming from, especially when your first inclination might be to disagree. Good academic *writing* can also be seen as a conversation between the writer and others who have thought about similar questions. So this way of doing things will carry over into the way we approach writing assignments as well.
5. To participate in and contribute to the common activities of the Core Human Questions Cluster of the Montserrat program.

Texts

The texts for the course are:

1) Homer, *The Essential Odyssey*, translated and edited by Stanley Lombardo, Hackett Publishing.
2) Twain, Mark, *Adventures of Huckleberry Finn.* Any unabridged and unexpurgated edition is OK; an inexpensive Dover republication will be available in the HC bookstore.
3) Strayed, Cheryl, *Wild: From lost to found on the Pacific Crest Trail*, Vintage Paperback.
4) Hardy, G.H. *A Mathematician's Apology*, with foreword by C.P. Snow, (Canto Classics) Cambridge University Press.

5) Euclid, *Elements*, Book I, online at:

`http://aleph0.clarku.edu/~djoyce/java/elements/elements.html`

6) Lakatos, I. *Proofs and Refutations*, Cambridge University Press.
7) Davis, P. and Hersh, R. *The Mathematical Experience* Study Edition, Birkhauser, 2012.

See course schedule for more information about when will be looking at each one.

Course schedule

A detailed day-by-day course schedule and listing of required and suggested outside events will be maintained on the course homepage (and will be accessible through the Moodle course management system). That listing is a tentative, evolving schedule, so it may change and you will probably want to refer to it frequently. Any important changes will also be announced in class well in advance.

Assignments and grading

1) *Midterm exam* (15 % of course grade) – tentative date: Friday, October 30.
2) *Final exam* (20 % of course grade) – the final will be given at the regular time for MWF 9 am classes, when that is determined.
3) *Writeups from group project days and problem sets* (about 5 assignments – 15 % of course grade)
4) *Three roughly 3-5 page papers*, and other shorter writing assignments. (Information and guidelines to be distributed later) (30 % of course grade)
5) *Presentation*: During our reading of the *Elements* of Euclid, each student, working with a classmate, will prepare an oral presentation, approximately 15-minutes in length, on one of the Propositions in Book I, as assigned by Professor Little (10 % of course grade)
6) Class participation (10 % of course grade)

I will be keeping your course average in numerical form throughout the semester, and only converting to a letter for the final course grade. The course grade will be assigned according to the following conversion table (also see Note below):

- A – 94 and above
- A- – 90 - 93
- B+ – 87 - 89
- B – 84 - 86
- B- – 80 - 83
- C+ – 77 - 79
- C – 74 - 76
- C- – 70 - 73

- D+ – 67 – 69
- D – 60 - 66
- F – 59 and below.

Note: Depending on how the class as a whole is doing, some downward adjustments of the above letter grade boundaries may be made. No upward adjustments will be made, however. (This means, for instance, that an 85 course average would never convert to a letter grade of B- or below, although it might be a B+ in some circumstances.) If you ever have a question about the grading policy or your standing in the course, don't hesitate to ask me.

Advice on how to succeed in this class

A good "work ethic" is key. As you should be able to tell from the course description above, you do not need to be a "math genius" to do well in this course. But you will need to put in a consistent effort and keep up with the reading and assignments.

Come to class. Unless you are deathly ill, have a genuine family emergency, are away at a game or meet of a college athletic team, etc. plan on showing up here at 9:00 am every Monday, Wednesday, and Friday this semester. Many of the class meetings will be structured around discussions or student presentations. Your participation is expected and needed for the success of the course!

Take notes and use them. This may seem obvious, but it is worth saying! Used intelligently, your notes can be a valuable resource as you work on problem sets and prepare for the exams.

Use the texts and class notes actively. Reading about mathematics is not like reading a novel. You will probably need to read and think over things more than once. You may want to work through examples to understand some of the topics that we do.

Set up a regular study schedule and work at a steady pace. It's not easy to play catch-up in a mathematics course (even when the course is part of a first-year program with additional goals beyond the mathematics). You should expect to budget at least 6 hours in a typical week for work outside of class. The best way to use your time is to do a few problems, some reading from the books, and reviewing of class notes every day.

Most importantly, if you are having difficulty learning something, get help as soon as possible. You can do this by asking questions during class (any time something isn't clear), or seeing me during office hours.

Statement on academic integrity

All education is a cooperative enterprise between teachers and students. This cooperation works well only when there is trust and mutual respect between everyone involved. To be become an engaged and advanced learner, you must be able to think and work both independently and in concert with your peers. The College academic honesty policy states: "As an institution devoted

to teaching, learning, and intellectual inquiry, Holy Cross expects all members of the College community to abide by the highest standards of academic integrity. Any violation of academic honesty undermines the student-teacher relationship, thereby wounding the whole community. The principal violations of academic honesty are plagiarism, cheating, and collusion.

Plagiarism is the act of taking the words, ideas, data, illustrative material, or someone else's statements, and presenting them as one's own without full and proper acknowledgment.

Cheating is the use of improper means or subterfuge to gain credit or advantage. Forms of cheating include the use, attempted use, or improper possession of unauthorized aids in any examination or other academic exercise submitted for evaluation; the fabrication or falsification of data; misrepresentation of academic or extracurricular credentials; and deceitful performance on placement examinations. It is also cheating to submit the same work for credit in more than one course, except as authorized in advance by the course instructors.

Collusion is assisting or attempting to assist another student in an act of academic dishonesty. The full statement on Academic Honesty in the College Catalog is available at

`http://www.holycross.edu/catalog/acad_program.pdf`

If you do not know how to correctly cite reference materials, consult with your professor, the campus Writers Workshop, or visit one of the links below:
Holy Cross Department of History:

`http://academics.holycross.edu/history/academichonesty.htm`

Wadsworth Cengage, "Plagiarism Prevention Zone"

`http://college.cengage.com/english/plagiarism_prevention.html`

NOTE: If in doubt about what you plan to do or write violates academic honesty, PLEASE ASK! The temptation to engage in an act of academic dishonesty may arise, but the chance possibly to enhance a single grade is not worth the loss of your personal integrity.

Specific guidelines for this course

In this course, all examinations will be closed-book. No consultation of outside sources of information or sharing of information with other students in any form will be permitted during exams. In group discussions, close collaboration with the other members of your group is expected. For the papers, please refer to the specific guidelines for each assignment regarding what is acceptable and how to cite sources other than the course texts if that is allowed. In general, some discussion of ideas for your paper with classmates and with Prof. Little will be acceptable as you prepare to write. But your final written work should be your own and any ideas you use from other sources should be acknowledged. On individual problem sets, discussion of the questions with other students in the class and with me during office hours is allowed, *even encouraged*. However, your final problem solutions should be prepared individually and the wording and organization of your final problem solutions should be entirely your own work. Moreover, if you do take advantage of any of

the above options for discussion of problems with others, you will be required to state that fact in a footnote accompanying the problem solution. Failure to follow these rules will be treated as a violation of the College's Academic Integrity policy.