MONT 105Q – Mathematical Journeys
Discussion – Using the Standard Normal Table
March 21, 2016

Background

Right-hand tail probabilities for a standard normal random variable (i.e. normal distribution with $\mu = 0$, $\sigma^2 = 1$) are given in the table on the attached sheet. For instance, The entry for $z = 1.43$ is .0764. This means that

$$P(Z > 1.43) = .0764.$$

From this we could also derive facts such as

$$P(Z \leq 1.43) = 1 - .0764 = .9236.$$

Moreover, by the symmetry of the standard normal curve, if $c > 0$, we can see

$$P(0 < Z < c) = \frac{1}{2} - P(Z > c) = P(-c < Z < 0).$$

Hence, for instance,

$$\begin{aligned} P(-1.0 < Z < 2.0) &= P(-1 < Z \leq 0) + P(0 < Z < 2) \\ &= \left(\frac{1}{2} - P(Z > 1)\right) + \left(\frac{1}{2} - P(Z > 2)\right) \\ &= (.5 - .1587) + (.5 - .0228) \\ &= .3413 + .4772 = .8185. \end{aligned}$$

For today's discussion problems, the following key fact will be needed too.

Key Fact: If Y is normal with mean μ and standard deviation σ, then

$$Z = \frac{Y - \mu}{\sigma}$$

is standard normal, and the table can be applied to Z.

For Example: Say we know that Y has a normal distribution with mean $\mu = 12$ and SD $\sigma = 2$. Suppose we want to know the probability $P(Y > 11.3)$. We can determine this from the Key Fact as follows:

$$\begin{aligned} Y > 11.3 \Leftrightarrow Z = \frac{Y - 12}{2} &> \frac{11.3 - 12}{2} = -.35, \text{ so} \\ P(Y > 11.3) &= P(Z > -.35) \\ &= 1 - P(Z > +.35) \text{ by symmetry} \\ &= 1 - .3632 \text{ from table} \\ &= .6368 \end{aligned}$$

In today's discussion, you will practice using the standard normal table to answer questions about normally distributed quantities following similar methods.

Discussion Questions

A) Let Z be a standard normal.

1) Find $P(Z > 1.29)$.
2) Find $P(Z < 1.0)$.
3) Find $P(-2.13 < Z < -0.56)$.
4) Find c such that $P(Z > c) = .05$. (You'll need to "interpolate" here. Do you see why?)

B) Y is normally distributed with mean 6 and SD 4. Find

1) $P(Y < 7)$.
2) $P(Y > 8.5)$.
3) $P(5 < Y \leq 8)$.

C) SlimMints are sold in two-packs with a stated label weight of 20.4 grams. The actual weights of the packages are normally distributed with mean $\mu = 21.37$ and SD $\sigma = .4$.

1) Let Y be the weight of a single package selected at random from the production line. What is the probability $P(Y > 22.07)$?
2) Suppose that 15 packages are selected independently. Let X be the number that weigh less than 21 grams. What is $P(X = 2) =$ the probability that exactly two have weight less than 21 grams?

We'll compare notes and I'll give the correct answers at the end of the class.