

MONT 104Q – Mathematical Journeys
Group Discussion Day 3
October 26, 2015

Background

As you can now appreciate from you have seen and read, Book I of the *Elements* of Euclid presents mathematics in a finished and very polished form. Recall that it was used from the start as a mathematical textbook, so the intent was certainly that people would learn geometry by reading it. However, note that Euclid does not attempt to address the question: *How did any one find these results and their proofs?* One might even say that the question is not relevant for him. What is relevant is the logical connections of the proofs leading up to the big result in Proposition 47.

So one possible criticism of Euclid is: *reading the Elements does not (in and of itself) prepare a student to find her/his own new theorems and proofs.* To be fair, it's pretty clear that that was not even the primary goal that Euclid had in mind! The ultimate purpose of the study of geometry for Plato, for instance, was *not to find new theorems*, but to *lead the soul to the contemplation of the eternal truth of statements about the world of geometric forms* as preparation for the study of philosophy.

This leaves a major question: How do we *figure out* what is true in mathematics and then find proofs of those statements? Today we will look at an exercise developed by the 20th century American mathematician Hassler Whitney, who later in his career developed a strong interest in reforming mathematical education to include more teaching designed to develop students' skills for *discovering patterns* and then *justifying* what they found.

Whitney was a big believer that “one of the best things a teacher can do is arrange questions or challenges so that a student can stumble upon something unexpected and feel a sense of self-empowerment as a result” (quoted from an as yet unpublished biography of Whitney by another contemporary mathematician named Keith Kendig). Many times that involves not telling them too much and keeping rules and directions to a minimum. So here goes:

Directions

1. Identify the pattern(s) in, and fill in the rest of, the table on the back of this sheet. Each square will contain a single whole number.
2. What number(s) go in the shaded boxes down the middle row and column, *and why?*

(Comment for 2: One possible way to answer this is to find a formula giving the numbers in all of the boxes. It may help to number the rows and columns in the table somehow and use the row and column numbers to develop your formula.)