Mont 104Q – Mathematical Journeys: From Known to Unknown Reading Questions for G. H. Hardy, A Mathematician's Apology

Note: In the Cambridge University Press editions, Hardy's long essay (starting on p. 61) is preceded by a Foreword written by the chemist and writer C.P. Snow. You are not responsible for that Foreword, although it's not long and you may find it interesting to read that as well if you have the time. It definitely sheds some interesting sidelights on Hardy's personality and life history.

- 1. Look up the word *apology* appearing in Hardy's title. The most commonly-used meaning today is something like "an expression of regret for harm one has done to someone or something." Is that the meaning that Hardy is using? If so, what does he have to apologize for? If not, what *does* he mean by this word, and have you read (or do you know of) any other *apologies* like this one?
- 2. What does Hardy say about the "function of a mathematician?" What does he say about "men who make" versus "men who explain" (or, for that matter, "men who teach")? Does that seem like a reasonable point of view to you?
- 3. According to Hardy, what are the principal motives that people have for pursuing a life devoted to research in an area like mathematics? (This would apply too, by the way, to many researchers in science and medicine!) Is that a recipe for a rewarding life? Does Hardy seem like someone who is satisfied with his own life, for the most part? (Some context: This work was written in 1940 when Hardy was 63 years old. He's definitely in "looking backward mode" near the end of his journey. He died in 1947.)
- 4. We'll start our study of actual mathematical proofs by studying the two main examples Hardy gives of "beautiful" mathematics in section 12, so don't worry too much about understanding every detail now. But what exactly are the statements being proved? Do you know these facts? Have you seen these proofs before?
- 5. Would Hardy say mathematics is closer to the sciences or to the humanities or the arts? Does this surprise you? And for Hardy, what is the ultimate criterion for whether a piece of mathematics is "good" or not? What are other qualities of serious, good mathematical work? Why do you suppose the idea of *proof* plays such a big role for him?
- 6. Probably the most controversial aspect of Hardy's ideas expressed in this book is his point of view about "useful" mathematics versus "useless" mathematics. Without judging it first, try to outline exactly what he says about that distinction for yourself. Then feel free to judge it if you want. What's the point of doing something that is totally "useless?"
- 7. What is the ultimate justification Hardy gives for the life he has chosen? Is that a sufficient reason to say one has led a good life?
- 8. Hardy states on page 101 that "There is no doubt at all, then, of the 'seriousness' of either theorem from section 12. It is therefore the better worth remarking that

neither of them has the slightest 'practical' importance." What evidence does Hardy give to justify this statement? The *RSA public key encryption system* is what almost all of today's web browsers use to make secure transmission of information over the internet possible. Look this up and try to identify the role of prime numbers in RSA. Was Hardy right about the lack of practical importance of the fact that there are infinitely many primes? How do you suppose he might feel if he knew that people are sending messages encrypted with this technology over the internet for all sorts of nefarious purposes (such as communication between members of hate groups, terrorist cells, groups of hackers, etc.)?