MONT 104Q – Mathematical Journeys: From Known to Unknown
Proofs from Class on Friday, October 2

Here are writeups of the various proofs the groups presented in class today on the problems from Wednesday's handout so that everyone has a "clean record" of what we did. The figures are on the back of this page.

B) Let ΔABC be an equilateral triangle and let D be a point on the side BC different from B, C. Prove that the line segment AD is shorter than the sides of the triangle.

Proof 1: (by contradiction). Assume AD is greater than the sides AB, AC, BC of the equilateral triangle ΔABC. By Proposition 18, $\angle ACD > \angle ADC$ and $\angle ABD > \angle ADB$. By "Common Notion 6," this implies $\angle ACD + \angle ABD > \angle ADC + \angle ADB$. By Proposition 13, $\angle ADC + \angle ADB =$ two right angles. So $\angle ACD + \angle ABD >$ two right angles. But that contradicts Proposition 17. Therefore AD is shorter than the sides of the triangle.

Proof 2: (also by contradiction). Assume AD is greater than the sides AB, AC, BC of the equilateral triangle ΔABC. Then by Proposition 16, the exterior angle $\angle ADB$ to triangle ΔADC is greater than the interior angle $\angle ACD$. By Proposition 18, in the triangle ΔABD, we have $\angle ADB < \angle ABD$ since the side $AD > AB$ by assumption. But Proposition 5 implies $\angle ABD = \angle ACD$ since $AB = AC$. This is a contradiction because we have $\angle ADB > \angle ACD = \angle ABD > \angle ADB$. (The angle $\angle ADB$ cannot be greater than itself.)

C) Prove the following important fact: Given a line and a point P not on the line, the perpendicular from P to the line (the result of the construction from Proposition 12) is the *shortest* line segment joining P to any point on the line. (*Note*: You will probably be tempted to use the Pythagorean Theorem here. We have not proved that!)

Proof 1: (This is the one I saw first and I think this is where Isabel was headed – another proof by contradiction): Assume $PC \geq PQ$ (or equivalently $PQ \leq PC$). Then in the triangle ΔPQR, by Proposition 18, we have $\angle PQC \geq \angle PCQ$. But then $\angle PQC + \angle PCQ \geq 2$ right angles. This contradicts Proposition 17.

Proof 2: (This is what Katie was saying at the end.) A direct proof (not by contradiction): Since $\angle PCQ$ is a right angle, by Proposition 17, $\angle PQC$ is less than a right angle. (If it weren't less, then it, plus $\angle PCQ$ would be more than two right angles and that contradicts Proposition 17). But since $\angle PCQ > \angle PQC$, Proposition 19 implies that $PQ > PC$.

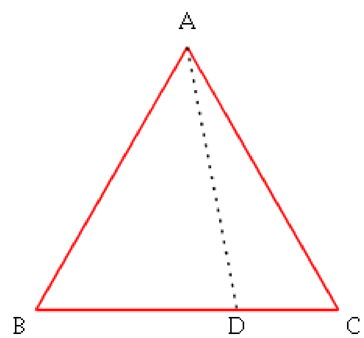

Figure 1: Figure for Question B

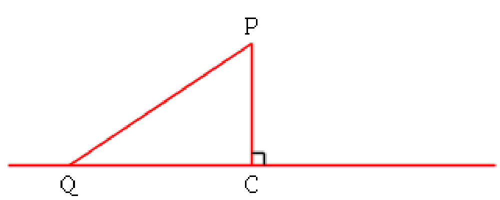

Figure 2: Figure for Question C