Mathematical Journeys: First day, 9/2/15

CHQ Theme:

How then shall we live when the journey may be as important as the origin or the destination?

While many other species (think migratory birds or monarch butterflies, for instance) regularly undertake almost incredible travels, human beings are quite distinctive too for the extensive journeys we have made. From our origins in Africa, over the course of tens of thousands of years, we have spread over the whole earth and adapted to extremely different habitats with the help of technology that allows us to journey ever faster and to new places. Our species has in fact been molded and transformed by those journeys and the history of those movements is preserved in our genetic diversity and the variability of our features. We have even set foot on the Earth's Moon and are perhaps poised to move farther out into our solar system if we can muster the will and the resources to do so.

On another level, too, every human life is a journey. With luck, we start out from a safe home with caring parents and a loving family. We gradually make our way into wider and wider communities; we have all sorts of different experiences; we grow in some ways, or perhaps we do not. And after many years (if luck is still with us), we reach that one destination that we must all experience eventually. Looked at this way, of course, you might say that our personal journeys are *everything* to us – they are our whole lives. But this means that the first part of our cluster theme, the "How then shall we live?" is the most basic and important question we all must ultimately address if our lives are to have meaning and purpose.

Many of the experiences we have along the way are also journeys of one sort or another. Those experiences can be some of the most influential ones we have because they can lead to conscious or unconscious choices of how we live. So in the CHQ cluster of Montserrat this year, through our common readings (this semester: Homer's *Odyssey*, Mark Twain's *Adventures of Huckleberry Finn*, and Cheryl Strayed's *Wild*) and our co-curricular activities, all of our seminars will be addressing this theme of the journey. Our first common reading (Homer's *Odyssey* next week) is really the prototype of the journey story in our literature, so it's a perfect place to start. The second cluster common reading, *Huckleberry Finn* by Mark Twain, has been seen as the source of most of distinctively American literature. We will see how Huck and Jim's trip down the Mississippi River in pre-Civil War America is another sort of epic journey. *Wild*, our final common reading, is a contemporary story of a woman's attempt to hike the Pacific Crest Trail solo, to find herself and put her life back together. All of these focus on the experience of journeys (often more than origins or destinations) and the way they can lead us to address that core human question: "How then shall we live?"

Right about now, I know some of you might be thinking – "What's up with all this?? I thought this was supposed to be a mathematics course!" (Be honest!) Well, the short answer is that this *will be*, partly, a mathematics course, but it will not be *only* a mathematics course, and it will almost certainly be different from any mathematics course you have taken before. After all, it's a part of the Core Human Questions Cluster of Montserrat! We'll be looking at one particular sort of mathematical work that ties into our theme in what I hope will be a challenging and thought-provoking way.

Let me try give you some initial ideas about what I have in mind. Mathematics *can be taught* as a collection of rote procedures for calculating things without much (or even any) indication why the procedures work or what they mean. And you probably have experienced this, since on some level, this is actually the way the subject is often taught, especially in grade school and high school! At the same time, calculators and mathematical software have become important tools because they can do almost any low-level operation millions of times faster and more accurately than any human being. So it's perhaps understandable that we have introduced them early into mathematical education, sometimes without the underlying knowledge needed to make sense of the masses of numbers they produce. But the experience of being turned into an unthinking and uncomprehending button-pusher (and then being severely penalized and made to feel stupid for making the almost inevitable resulting small errors in carrying out processes that are not really understood) is not a pleasant one for most students. So if you asked most people in the US today which subject in school they liked the least, mathematics would win hands down. This is especially discouraging for those of us who love the subject because most of us would agree that those rote calculations are *not what the subject is really about anyway!*

So, if that's not what mathematics is about, what is it about? Well, I would say math is really about finding patterns in quantity (numbers) and arrangements (geometry), about creatively *figuring things out*, ultimately about learning to use the power of our brains and not just accepting things without justification. In fact, the word "mathematics" itself comes from words related to the Greek verb $\mu\alpha\nu\theta\acute{\alpha}\nu\omega$ ("to learn by inquiring," or sometimes "to understand.") So here's one possible (rather extreme) answer to our question "How then shall we live?" – Don't accept anything on faith. Always ask why? how? who benefits? Warning: This is much harder than it might sound and it can make you very unpopular.

If you think about it, if this is what mathematics is really about, then we also need to have ways to convince others that the things we have figured out, or that we think we understand, are actually true. And this is where the idea of *proof* comes into play. At the foundation level, a proof is an argument that a statement is true. It should *convince* the reader or hearer – so in a sense someone who discovers something new in mathematics, and then the reader or hearer who learns about it, has to experience a journey from one state of knowledge to another. The origin is what we knew before, the destination is something new (previously unknown), and the proof can be seen and experienced as a sort of journey. At its heart, this isn't so different from persuasive speaking or writing in many other areas of human thought (including some of the writing we will be doing for this seminar!). So it's perhaps not so surprising that the people who first experimented with the ideas of democratic self-government (where citizens had to convince one another about the right actions to take) and trial by jury for deciding law cases – the ancient Greeks – were also heavily involved in developing the idea of mathematical proof.

After reading the *Odyssey*, we'll start off by looking at what a well-known 20th-century English mathematician, G.H. Hardy, said about his life and the subject of mathematics in his book *A Mathematician's Apology*. Hardy's life story raises a number of interesting questions about "How then shall we live?" In his book, Hardy also highlights the way ingenious proofs can be seen as the lifeblood of mathematics. He will lead us to the major focus of the first section of our course. Much as the *Odyssey* has served as a prototype for stories about journeys, the *Elements* of Euclid has served as a prototype for textbooks of mathematics, right down to the present day. As we will see, the section we will study in detail – Book I – is a very tightly-organized sequence of theorems and proofs leading to a wonderful proof of a familiar result from plane geometry that all of you have seen (although you may not know a proof for it).

The second mathematical portion of the course will be devoted mainly to the question: How do we *find proofs*? We will have seen by this point in the course that this step in the process is not addressed at all in Euclid's work. Many, many generations of students have found this frustrating, to say the least. And indeed, having only the ultimate destination or *finished product* in the form of a polished proof, *à la Euclid*, is arguably not the most productive way to learn mathematical ideas, although teachers of mathematics have different opinions about what might replace it(!) In this part of the course, we will look at the "messier" cycle of conjectures (informed guessing), attempted proofs, counterexamples, and refutations by which actual proofs come into being through the interaction of groups of real people.

If any of you go on to major in mathematics, you will see that upper-level books and courses start to look an awful lot like what we're going to be doing in this course, though with more complicated and intricate patterns and proofs to find and to learn. I hope some of you will do that, but even if this is the last (even partly) mathematics course you ever take, I hope you find it to be a mind-changing experience and that you gain a better appreciation for what the subject is all about.