As we get close to the end of our (mathematical) journey together, I hope you'll take the opportunity to think back and reflect about all the different experiences we have had this year and this semester in particular.

Just this term, we studied Charles Darwin's record of one of the most important journeys in the history of science – his 5-year voyage of exploration in the 1830's as a civilian traveling with the crew of the British warship *H.M.S. Beagle*. Darwin's purpose on this trip was to make measurements of longitude, and observations of the flora, fauna, and geology of various places in South America, various Pacific Islands, etc. gather samples, and describe them in writing as the ship circumnavigated the globe. What he saw on the way influenced his thinking and formed the nucleus of the revolutionary ideas on evolution by natural selection that he published in his later work *The Origin of Species*. Perhaps Darwin's most important achievement on this trip was to begin looking beyond received creation stories. He opened himself to what the physical evidence the actual world was showing him about the age of the earth, the long time spans required for changes in landscapes and evolution of life forms. This was just one of the momentous advances in science in the 19th century and I would say we (i.e. humanity as a whole) haven't completely come to terms with some of its implications even now.

Somewhat analogous journeys of discovery were also underway within mathematics at roughly the same time. For instance, the "undiscovered country" of non-Euclidean geometries was brought to light through the work of Janos Bolyai, Nikolai Lobachevsky, and Carl Friedrich Gauss. This development had the consequence that the propositions of Euclidean geometry could no longer reasonably be taken as self-evidently truths about the physical universe. This called into question just what Euclidean geometry was doing and it opened the way for explorers (like Albert Einstein, Stephen Hawking, and many others) to try to understand the actual geometry of physical space approaching that question in the same spirit of openness to the actual evidence of the universe that Darwin developed.

Somewhat later, motivated by disturbing questions about the theory behind differential and integral calculus, mathematicians like Georg Cantor and others attempted to lay new groundworks for mathematics in the theory of sets. Eventually, as we saw in reading *Logicomix*, troubling results like Russell's Paradox emerged that called this whole enterprise into question and threw the subject into a pretty severe *crisis of foundations*. By the early 20th century, this train of thought led to a realization that the strict axiomatic, deductive method exemplified by Euclid's *Elements* has definite limitations. Specifically, the *Incompleteness Theorems* proved by Kurt Gödel says, essentially, that within any axiomatic system powerful enough to encompass the arithmetic of the natural numbers, there will always be *true* statements that *cannot be proved within the system*, and the consistency of the system (i.e. it's freedom from internal contradictions) is one of the unprovable statements(!)

Considering the level of controversy it produced and the time and effort expended by many mathematicians and logicians trying to resolve it, it is perhaps surprising that the ultimate outcome of the crisis in foundations has been pretty *inconclusive*, to say the least! None of the solutions proposed to give mathematics provably correct foundations has been entirely successful although they have continued on a small scale in more and more technical sub-fields of logic! As a result, we do not really know whether the mathematics we have is *consistent*, and even worse, that sort of consistency is essentially *undecidable* within mathematics itself (i.e. there is no way to prove whether or not our mathematics will eventually produce contradictory results!) Yet pure mathematicians have gone on proving new theorems all the same. Applied mathematicians use those tools more than ever and they appear to "work" extremely well as descriptions of all sorts of real world phenomena.

Where does this leave us? A recent (and, to me, very convincing) point of view is that the most productive future developments of mathematics will lie, not in the vast abstractions and (illusory) certainty of Euclidean-style axiomatic mathematics, but rather in a journey toward a deeper understanding of how *chance and randomness* influence real-world processes. This is the message of a rather controversial article called *The Dawning of the Age of Stochasticity* by David Mumford that we read a few weeks into the semester. Part of Mumford's agenda there is to say that a probabilistic orientation might even clarify some continuing conundrums in the foundations of mathematical logic like the Continuum Hypothesis (one of the Hilbert problems presented at the 1900 Paris Congress of Mathematicians). This is the statement that are no cardinal numbers between the cardinality of the natural numbers – often denoted $\aleph_0$ after Cantor's notation – and the cardinality of the real numbers. This, and its negation, are both known to be equally consistent in combination with the now-standard, revised ZFC set theory – a famous result by Paul Cohen in the 1960's. However, Mumford explains a "thought experiment" by a logician named Chris Freiling that leads to an intuitively appealing answer [describe]. But those ideas are currently still very much "under review" and I think the prevailing opinion is that there are "issues" with Freiling's approach too.

There is no doubt that statistical thinking is opening up whole new fields of inquiry and applications of mathematics such as

- *bioinformatics* – unlocking the DNA "code of life" in individuals, with exciting possible medical applications
- *mathematical modeling* – of organisms, populations, larger-scale systems like the earth's climate
- *"big data"* – Finding patterns in the huge amounts of information that we can handle with new technologies

Scientists and all of us are involved in a huge journey from the unknown to the known. We have seen some of the basics in our study of probability and statistics.

No matter how much we learn about the world and its mathematical underpinnings, though, I think the human mind and its mysteries (where do our emotions of love, or reverence for nature or God, or hatred come from? why are we capable of both great self-sacrifice, artistic achievements and unimaginable violence and cruelty?) will always remain another "undiscovered country" that we all have to come to terms with. We all have to take journeys of exploration to begin to understand ourselves as individuals and humanity as a whole. Even when they haven't connected directly with our mathematical topics, our CHQ texts and activities like *the Curious Case of Benjamin Button*, *Hamlet*, the films *Incendies* and *The Music Box*, the MFA trip, etc. have aimed to help you take the first steps on those journeys of discovery.