MONT 104Q – Mathematical Journeys: From Known To Unknown
Working Within Euclid's System – September 30, 2015

Background

By this time we have covered up to Proposition 16 in class, and you have read through Proposition 22 in Book I of the *Elements*. To recap, recall that we know the Common Notions and Postulates, a number of basic constructions, as well as the following facts:

Proposition 12: Given a line and a point not on the line we can construct a perpendicular from the point to the line.
Proposition 15: Vertical angles at the intersection of two straight lines are equal
Proposition 16: An exterior angle of a triangle is greater than either nonadjacent interior angle
Proposition 17: The sum of each pair of interior angles in a triangle is less than two right angles (i.e. less than 180° for us).
Proposition 18: If in a triangle one side is greater than another side, then the angle opposite the greater side is greater than the angle opposite the other side.
Proposition 19: If in a triangle one interior angle is greater than another interior angle, then the side opposite the greater angle is greater than the side opposite the other angle.
Proposition 20: (the "*triangle inequality*") The sum of any two sides of a triangle is greater than the other side.

Even though Euclid never quite states this, he uses and we will also use an additional "common notion" today:

("CN 6") *If one thing is greater than a second and equals are added to both, then the first sum is greater than the second sum.*

Side trips on our main journey

Using *only* (I repeat, *only*!) what we know at this point, prove the following, with justifications for every step.

A) (A "warm-up") Using Proposition 16, prove Proposition 17. (Hint: Let the triangle be ΔABC. "Without loss of generality" we can consider the two interior angles at A and B. Extend the side CB to CD and apply Proposition 16 to the exterior angle $< DBA$.)

B) Let ΔABC be an equilateral triangle and let D be a point on the side BC different from B, C. Prove that the line segment AD is shorter than the sides of the triangle.

C) Prove the following important fact: Given a line and a point P not on the line, the perpendicular from P to the line (the result of the construction from Proposition 12) is the *shortest* line segment joining P to any point on the line. (*Note*: You will probably be tempted to use the Pythagorean Theorem here. We have not proved that!)