MONT105Q – Mathematical Journeys
Problem Set 2
due: Friday, March 4, 2016

I. Four cards will be dealt off the top of a well-shuffled deck (without replacement, of course!).

(A) Which is more likely, or are they equally likely? Explain and/or calculate probabilities:

(i) The cards are a spade, then a heart, then a diamond, then a club, or

(ii) the cards belong to four different suits?

(B) Again, which is more likely and explain:

(i) The cards all belong to the same suit, or

(ii) the cards belong to four different suits?

II. One ticket will be drawn at random from a box containing 6 tickets: three with the numbers 2,2,3 on blue paper, and three with the numbers 2,2,3 on green paper. Are the number and the color independent? Why or why not?

III. A fair, standard die is rolled 10 times. Thinking of the results as *ordered* sequences of numbers 1,2,3,4,5,6, find the probability of

(A) getting 10 sixes

(B) not getting 10 sixes (that is, anothing other than 10 sixes)

(C) all rolls showing four or fewer spots

(D) the sum of the numbers shown in the 10 rolls is greater than or equal to 57.

IV. The probability of an event A is $1/4$ and the probability of a second event B is $1/5$. True or false, and explain:

(A) If A and B are independent, they must be mutually exclusive.

(B) If A and B are mutually exclusive, then they cannot be independent.

(C) If the probability of $A \text{or} B$ is $9/20$, then A and B are mutually exclusive.

(D) If A and B are independent, then the conditional probability $P(A|B)$ is $1/4$.

V. A fair coin will be tossed 10 times. What is the probability that exactly 2 of the first 5 tosses will yield heads, and exactly 3 of the last 5 tosses will yield heads? What are you assuming here? Also, *without calculations, explain* whether your answer is less than, greater than, or equal to the probability that there are exactly 5 heads in the 10 tosses.

VI. A (very) simple keno game works like this: You make a \$1 on a single number. If your number comes up you get your bet back and \$2 more. If not, you lose your bet. You have a 1 in 4 chance of matching your number.

(A) State a box model that is equivalent to the outcome of this game. (That is, the tickets should tell the dollar amounts you win or lose and a draw from the box should be the same as playing one round of the game.)

(B) What is the expected value of the amount you win (or lose) in one play?

(C) How much would you expect to win or lose if you played 100 times?