MATH 110 – Selected Solutions for PS 1

1.3 / 5 (last part) To add with our base 10 number system, we need to know the digit sums $1+1, 1+2, \dots, 1+9, 2+2, \dots, 2+9, 3+3, \dots, 3+9, \dots, 8+8, 8+9, 9+9$. (There is nothing to do when adding 0, so those are not counted here.) There are $9+8+7+\dots+2+1 = \boxed{45}$ of these sums. It can be argued that the Egyptians did not need to memorize *any* of this, since they could just <u>count</u> and "carry" to the next digit: eg. $\overset{|||}{|||} + \overset{|||}{||||} = \cap\ |||$ (10 + 3 left over).

1.10 / 1 (a) The problem says use false position, but it was also OK if you thought about it this way: "Compute with $1+\bar{2}$ to get 16"

$$
\begin{array}{rcl}
1 \times (1+\bar{2}) & = & 1+\bar{2} \\
— \ 2 \times (1+\bar{2}) & = & 3 \ — \\
4 \times (1+\bar{2}) & = & 6 \\
— \ 8 \times (1+\bar{2}) & = & 12 \ — \\
— \ \bar{\bar{3}} \times (1+\bar{2}) & = & 1 \ —
\end{array}
$$

so the number is $8+2+\bar{\bar{3}} = 10+\bar{\bar{3}}$.

(b) The problem can be broken down like this in modern language: $(1+\bar{\bar{3}})x = y$ and $y - \frac{1}{3}y = 10$ what is x?

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \bar{\bar{3}}\, y = 10$

we can solve this in two steps:

first compute with $\bar{\bar{3}}$ to get 10

doubling

— $1 \times \bar{\bar{3}} = \bar{\bar{3}}$

— $2 \times \bar{\bar{3}} = 1 + \bar{3}$

— $4 \times \bar{\bar{3}} = 2 + \bar{\bar{3}}$

— $8 \times \bar{\bar{3}} = 5 + \bar{3}$

$\Rightarrow \quad y = 1+2+4+8 = 15$.

then $(1+\bar{\bar{3}})\,x = 15$ so proceed in similar way by doubling:

— $1 \times (1+\bar{\bar{3}}) = 1+\bar{\bar{3}}$ —

$2 \times (1+\bar{\bar{3}}) = 3+\bar{3}$ add to 15

$4 \times (1+\bar{\bar{3}}) = 6+\bar{\bar{3}}$

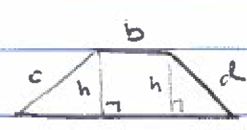
— $8 \times (1+\bar{\bar{3}}) = 13+\bar{3}$ —

so $x = 1+8 = \underline{\underline{9}}$.

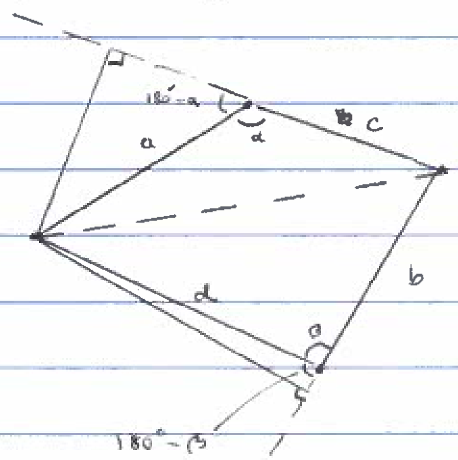
1.11 / 3

(a) In a rectangle  the opposite sides are equal, so the Egyptian formula gives $\frac{2a}{2}\cdot\frac{2b}{2} = ab$, which is the correct area.

(b) For a non rectangular parallelogram  the Egyptian formula gives ab again, but the actual area is $ah < ab$ since $h < b$.

(c) For a trapezoid  then Egyptian formula gives $\left(\frac{a+b}{2}\right)\left(\frac{c+d}{2}\right)$. the actual area is $\frac{1}{2}(a+b)\cdot h$. Both $c > h$ and $d > h$, so so $\frac{(a+b)}{2} \cdot \frac{(c+d)}{2} > \frac{1}{2}$ ca width again.

(d) (Treated as Extra Credit) Can the Egyptian formula $A \doteq \frac{(a+b)}{2} \cdot \frac{(c+d)}{2}$ even give the correct area for a quadrilateral that is not a rectangle? the answer is: NO. To show this, we'll use a correct modern form for the area, based on splitting the quadrilateral into triangles:



$$A = \frac{1}{2} ac \sin(180-\alpha) + \frac{1}{2} bd \sin(180-\beta)$$

$$= \boxed{\frac{1}{2} ac \sin(\alpha) + \frac{1}{2} bd \sin(\beta)}$$

If we split along the other diagonal, we get

$$A = \boxed{\frac{1}{2} bc \sin(\gamma) + \frac{1}{2} ad \sin(\delta)}$$

where γ is the angle between the sides of lengths b, c and similarly for δ.

multiplying each by $\frac{1}{2}$ and adding we also have

$$A = \frac{1}{4} ac \sin(\alpha) + \frac{1}{4} bd \sin(\beta) + \frac{1}{4} bc \sin(\gamma) + \frac{1}{4} ad \sin(\delta)$$

unless all four angles $\alpha = \beta = \gamma = \delta = 90°$, this is

$$< \frac{1}{4} ac + \frac{1}{4} bd + \frac{1}{4} bc + \frac{1}{4} ad = \frac{(a+b)}{2} \cdot \frac{(c+d)}{2}.$$