MATH 110 – Mathematics Through Time
Research Papers
October 27, 2013

General Information

As announced in the course syllabus, one of the assignments for the course this semester will be research. You will be working on this individually and the goals will be to prepare a paper of somewhere between 5 and 10 pages addressing a topic related to the historical questions we are studying in the course.

Schedule and Deadlines

- *On or before Friday, November 1* – Inform me *by email* which of the topics you want to work on. Since each project area has several different directions that might be pursued (see the descriptions below), write up a short paragraph giving a description of the aspect(s) you would like to work on. I hope to have a good mix of different topics represented, so please be open to suggestions for changes if it looks as though that is not developing.
- *Papers will be due no later than 5:00pm on Monday, November 25.*

Some Guidelines

The goals of this assignment are for you to collect information about your topic from the various sources you find, for you to learn and assess that information, and then for you to present your analysis of that information. The evaluation of your papers will be based on how well you have addressed the following guidelines and expectations:

- You should distill the results of your investigations into a central argument. A good research paper of this kind should be more than just a compilation of information from all the different sources you consulted. It should clearly show that you have thought independently about the information you found, that you have weighed the evidence for the various claims that were made in your sources, and that you have a central theme or argument about your topic that you want to present. It is certainly permissible to say you disagree with points of view presented in some sources, if you can explain why you think that and back up your opinions with appropriate evidence.
- The paper should be *well organized* and the writing should give the reader a clear indication where you are heading with your central argument at all times.
- Pay special attention to the first few paragraphs that will serve as an introduction. Catch the reader's attention, explain the significance of the topic or theme you will discuss. Say what you will do in general terms, without going into all the details from the start.
- Also pay special attention to the final few paragraphs, which will serve as a conclusion for your paper. Don't overstate the importance of your findings, and be honest if there are limitations. You might discuss how your investigations could be continued in further research.

- Give proper credit to sources you consulted that contributed to your ideas about the questions you studied. Use footnotes or endnotes to identify direct quotations from your sources, and also to indicate the sources that contribute to specific points you are making even when you are not using direct quotations. I don't care so much about the exact form of citations that you use as I do that you are following some standard convention with care.
- In a *References* section at the end, include all books, articles, and websites you used in the preparation of the work. For books, give the author(s), title, publisher, place and year of publication. For articles, give the author(s), title, journal name, volume, year, and pages. For any websites, give the full URL, the author (if that can be determined), and the date you consulted it.
- Be clear, concise, and correct in your writing. Aim for *no typos, misspellings, or grammatical problems*. But even more importantly, each paragraph should have a clearly evident purpose in relation to your main argument.
- Use figures, graphs, etc. sparingly in the main text. (If you want to include more of these, that can be done in an additional Appendix section at the end.)
- Proofread your work carefully and have an "impartial" reader or readers look at it and give you comments. This can be other students in the class or me. Be prepared and willing to *revise* your work based on the comments you get. Of course, this means that the writing must not be put off until the evening of November 24(!) Be sure you get started early enough so that the input can be put to productive use.

Project Ideas

Topic 1 – Eleanor Robson's Reinterpretation of Plimpton 322

As we mentioned briefly in class, Eleanor Robson is a current scholar of old Babylonian mathematics whose work has gone a long way toward putting that mathematics into a convincing historical and cultural context. She has provided, in particular, an interesting and novel "take" on what Plimpton 322 might have meant to its creators. See the link from our course homepage to an article she published on this in 2002 in the *American Mathematical Monthly*. If you were intrigued by the Old Babylonian Mathematics that we studied, this would be a very good topic to consider.

Topic Area 2 – Further consideration of Euclid's Elements

There is, of course, much more to say about Euclid. In particular, since we looked only at Book I, there are many different topics that could be formed by considering the contents of the remaining 12 books, II - XIII. For instance, students who have taken MATH 243 (the Algebraic Structures course), or others interested in number theory, will see that this side of mathematics is developed in Books VII, VIII, and IX, through properties of the prime numbers, the "Euclidean algorithm" for the greatest common divisor, and many other related topics.

Along a different line of inquiry, some historians have proposed that the overall goal of the *Elements* was to build up to constructions of the five *regular polyhedra* or *Platonic solids*

– the regular tetrahedron, the cube, the regular octahedron, and the regular dodecahedron and icosahedron. Euclid devotes Book XIII primarily to these figures and finishes with an argument that these five are the *only* regular polyhedra. The article by Bill Casselman on the course web site (from the Notices of the American Mathematical Society) is a good place to start if you are interested in that side of this.

Topic Area 3 – Jesuit Mathematicians, Astronomers, and Missionaries

In our discussion of non-Euclidean geometry, we will mention some of the contributions of Girolamo Saccheri, S.J., who tried to prove Euclid's fifth Postulate from the other four Postulates. A number of other Jesuit mathematicians, astronomers, and missionaries have been involved in various ways in other mathematical and historical topics we have touched on.

For instance, Christopher Clavius, S.J. was a geometer and astronomer from the early years of the Society of Jesus who played a major role in mathematics education in Europe and prepared an edition of Euclid suitable for use as a school textbook. Clavius was also a key figure in the development of our current Gregorian calendar system.

Matteo Ricci, S.J. (a student of Clavius) was one of the early Jesuit missionaries in China. He facilitated a rich scientific, calendrical, mathematical, and religious exchange between Europe and the last of the Ming dynasty emperors of China. (He was also well-known for his techniques of committing information to memory and a biography called *The Memory Palace of Matteo Ricci* by Jonathan Spence is a fascinating, though non-chronological, portrait of Ricci and his time in China.) Those contacts continued after Ricci's death through contacts between other Jesuits including Johann Schreck, S.J. and Johann Adam Schall von Bell, S.J. and emperors from the Qing dynasty.

Before entering China, Ricci also worked briefly in the Jesuit mission in Goa in south India, where other Jesuit mathematicians had contacts with Indian mathematicians. It was this connection that George G. Joseph posits as plausible evidence for transmission of Indian precursors of ideas from calculus from Kerala to Europe in the early 1600's in *The Crest of the Peacock* and other subsequent works. David Mumford's somewhat more technical review of the book *Mathematics in India* (authored by Kim Plofker) in a recent issue of the *Notices of the American Mathematical Society* may also be of interest, although it does not deal with the suggested "Jesuit connection." *The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook* is also an excellent resource for these topics. Caution, though: because of the level of the mathematics involved in this aspect of the topic (infinite series, Taylor series of functions, and so on), I would recommend a possible project about the suggested precursors of calculus topics in Kerala mathematics only for students who have already completed MATH 136 or 132 or the equivalent.

There is material for several distinct paper topics here – three different ones at least. The first step would be to narrow down the focus to something suitable for your paper. If you are interested in pursuing a part of this, I would be happy to discuss the options.

Topic Area 4 – Mathematics, the arts, logic and other connections

We will barely scratched the surface of the art of M.C. Escher in our discussion of his non-Euclidean tesselations of the plane. His symmetry drawings were just one

facet of his work, although many other aspects were also mathematically-inspired. One nice project topic would be to investigate some of the other mathematical aspects of Escher's work and the extent to which he was aware of and in contact with the work of professional mathematicians. To what extent was he really "doing mathematics" or "thinking mathematically" in his art? Several books and articles of Doris Schattschneider would be the best places to start for this. Her book *Visions of Symmetry* is a great source for a discussion of Escher's symmetry notebooks. Her article *"The Mathematical Side of M.C. Escher"* from a recent issue of the *Notices of the American Mathematical Society* is also very interesting.

I first learned about many of the ideas connecting Escher, Bach, mathematics, and so on from another wonderful book called *Gödel, Escher, Bach* by Douglas Hofstadter. We have not discussed who Kurt Gödel was in this course. Let me just say briefly that he was one of the great *logicians* of the 20th century. One of his most important contributions was a proof of a result called the Incompleteness Theorem. This work has had a tremendous influence on the way some people (mainly philosophers) think about mathematics. The Incompleteness Theorem says, in essence, that in *every axiomatic mathematical system* that is "powerful" enough to describe the basic properties of addition and multiplication in the integers, there are *true statements that cannot be proved to be true, and false statements that cannot be proved to be false.* (In a sense, you might think of this as a refutation of the whole idea of Euclidean mathematics – any system like Euclid's axiomatic development of number theory in the later books of the *Elements* is necessarily incomplete in the sense that deductive proof cannot uncover all the true statements about numbers.) The idea he used to derive this rather unexpected and unpleasant (at least for mathematicians) result was a clever use of a sort of *self-reference.* In intuitive terms, he showed there were ways to "encode" mathematical statements and proofs as integers, and hence that it is possible to work paradoxical statements like "this statement is false" (think about that one!) right into the "fabric" of the theory of numbers. Hofstadter discusses this in a very lively style, and draws connections between Gödel's proof and some Escher prints and Bach music. Hofstadter also proposes that perhaps this sort of self-reference might be the key to creating *artificial intelligence* – a hot topic back in the 1980's when this book was written, and an idea whose time may be coming again. It's a very "heady" mixture, but understanding it is definitely worth the effort.

Topic Area 5 – More on Islamic Mathematics

In class we will discuss some of George G. Joseph's ideas about the ways Islamic mathematicians united and developed the legacies of Greek (and Indian) mathematics during Europe's "Dark Ages" and then were instrumental in the reintroduction of those ideas to Europe during the Renaissance period. However, due to limitations of time, we will not be able to touch on a number of important parts of this historical arc. There are several different aspects of this story that would make for very good final project topics. First, the establishment of the library and translation center known as the Bait al-Hikma ("House of Wisdom") in Baghdad during the rule of the Abassid caliphs was instrumental in preserving the classic Greek texts for later scholars. There is a recent book called *The House of Wisdom* by Jonathan Lyons that deals with this. A slightly older book called

Aladdin's Lamp: How Greek Science Came to Europe Through the Islamic World by John Freely covers a lot of the same ground. Both of these books are quite complete, but somewhat dry, discussions of who did what and when they did it.

In addition to the scholarly center at Baghdad, the Islamic civilization of Al Andalus in present-day Spain became another great center of learning under the Umayyad caliphs and their successors. In particular, many of the translations of Greek classics into Latin (which hastened their reintroduction into Europe) were done in the city of Toledo in a rather amazing collaboration of Islamic and Christian scholars in the 1100's and 1200's. The two books mentioned above in this area deal with some of this story. Another book that deals with the context for this contact and with the story of Al Andalus in general is *The Ornament of the World* by Maria Rosa Menocal. The BBC TV series "When the Moors Ruled in Spain" from 2005 is another (but inevitably slightly superficial) treatment. This aspect of this group of topics would be great for anyone who wants to learn more about the culture that produced the Alhambra in Granada and its mathematical accomplishments.

The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook is also an excellent resource for the actual mathematics connected with these topics.

Topic Area 6 – More on Chinese Mathematics

In some cases, Chinese mathematicians discovered results and techniques that were only developed much later in Europe. Moreover, there were more extensive trade contacts between China and India, and between India and Europe than has sometimes been realized. So the whole development of our modern understanding of mathematics almost certainly involves more exchanges of ideas than than traditional histories often acknowledge. There are a number of slightly more technical mathematical topics from Chinese history that would be interesting for people who might want to explore this area. Two ideas in particular that might form the basis for good projects are

1. the result called the *Chinese Remainder Theorem* concerning simultaneous congruences modulo several different integers, and
2. Chinese techniques for solving simultaneous systems of linear equations that anticipate the modern ideas of matrix form for systems and Gaussian elimination

Either one of these (as well as several other possible mathematical topics) would be acceptable as a paper by itself. So there are again several different possible topics here. *The Crest of the Peacock* has good discussions of both of these, so that is probably the best place to start. *The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook* is also an excellent resource for these topics. In each case, the idea would be to explain the cultural and historical context for the mathematics, then work through the details of what the Chinese actually did, together with its relation to modern methods.

Topic Area 7 – A Topic of Your Choice

If there is another topic you would prefer to work on, I am open to suggestions. If you want to propose a topic of your own, you *must get my approval* before starting to work. For the November 1 deadline above, write up a short description of the topic or questions

you want to look at and how you want to try to address them. I will let you know as soon as possible whether you have my approval to proceed.