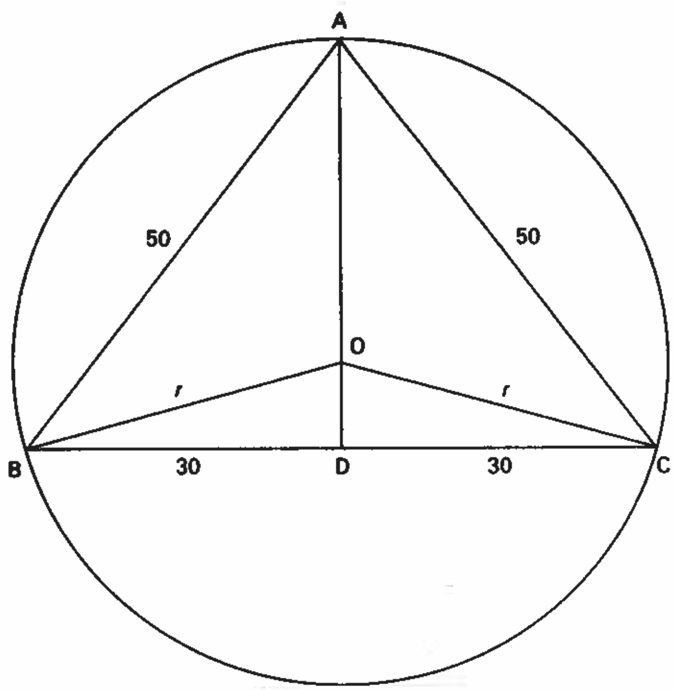
MATH 110 – Mathematics Through Time
Problem Set 2 – Babylonian Mathematics
due: Monday, September 23

I. It is now generally conceded that the Babylonians knew the general Pythagorean relation for right triangles, although no trace of a general proof has been found in any of their surviving work (and it is doubtful that they even attempted to articulate one). One of the strongest pieces of evidence for this is the following problem from an Old Babylonian tablet found in the city of Susa. The tablet is badly damaged, but it can be inferred from the parts of the diagram that remain visible that the problem was: *Determine the radius of the circumscribed circle for an isosceles triangle with sides* 50, 50, *and* 60:



(The modern labels have been added for the purposes of the problem. Also see the cover of our BJB text(!))

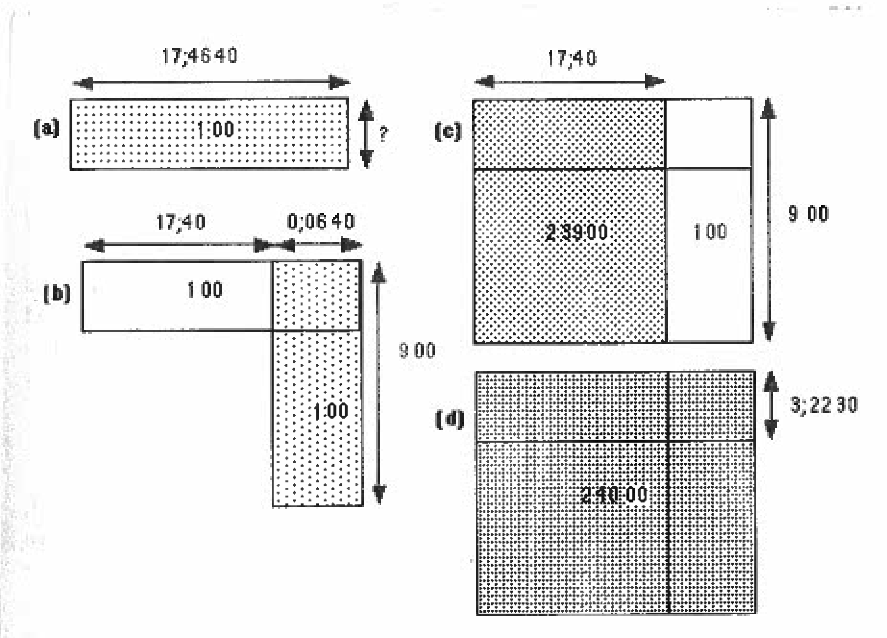
A) Use modern algebra and base-10 arithmetic to solve for the length $r = OC$.
B) What geometric facts are being used here? For instance, the procedure given on the tablet basically says: *break the 60 side in half at D and connect the top A of the triangle to D. The center of the circumscribed circle is then at O along the line AD.* What is true about the triangle ΔOCD? What must be true about the lengths OA, OB, OC?

II. Eleanor Robson has argued persuasively that the problem from YBC 6967 (and the information contained on Plimpton 322) is closely related to the question of computing "reciprocal pairs" x and $60/x$ (that is, ones more complicated than the ones in the standard table we saw in Discussion 1). The following is a literal translation of a tablet from the city of Nippur:

1. What is the reciprocal of 17; 46, 40?
2. You, in your proceeding: solve the reciprocal of 0; 06, 40. You will see 9; 00.
3. Multiply 9; 00 by 17; 40. You will see 2, 39; 00 come up.
4. Append 1; 00. You sill see 2, 40; 00.
5. Solve the reciprocal of 2, 40; 00. You will see 0; 00, 22, 30.
6. Multiply 0; 00, 22, 30 by 9; 00. You will see 3; 22, 30.
7. Your reciprocal is 3; 22, 30. That is the procedure.

A) Using base-10 arithmetic and modern notation, check each of the steps of the tablet and verify that the computations are correct

B) Robson has analyzed the process here in terms of the following cut and paste steps:



Explain how each part (a) to (d) of the figure corresponds to one or more steps in the procedure outlined above. What do the "multiply," "append," and "solve the reciprocal" steps correspond to in geometric terms? How does all of this relate to Hoyrup's analysis of YBC 6967?

III. Not all of Old Babylonian mathematics dealt with "geometric algebra" problems like the ones we have studied from YBC 6967, YBC 7289, etc. For example, an Old Babylonian tablet from about 1700 B.C.E (now held by the Louvre in Paris) has the following very practical problem: *Find how long it will take a certain sum of money to double itself at compound annual interest of* 20%. (Yes, they had compound interest even in this ancient civilization(!)) This means that you will have $1.2\times$ the original amount after 1 year, $(1.2)^2\times$ the original amount after 2 years, $(1.2)^3\times$ the original amount after 3 years, and so on. The question is: How many years will be needed until you have twice the original amount? (Fractional parts of years are also allowed.)

A) The Babylonian method of solution (written with base 10 numbers and in modern language) was this: First compute the powers to see that $(1.2)^3 = 1.728$ and $(1.2)^4 = 2.0736$. So the doubling will happen between the 3rd and 4th year. To find the doubling time, find the point on the straight line through $(3, (1.2)^3) = (3, 1.728)$ and $(4, (1.2)^4) = (4, 2.0736)$ with $y = 2$. The x-coordinate of that point is the doubling time. Carry out the calculations to find this time.

B) The Babylonian tablet gives the answer by this method as the base 60 number

$$(3; 47, 13, 20)_{60}$$

(with fractional part after the ;). Is this correct (does it agree with with you did in part A)?

C) Is this method exact or an approximation? Explain.

D) Solve the problem *exactly* by modern methods and compare with the Babylonian answer. How different are they? *Hint:* logarithms.