MATH 110 – Mathematics Through Time
Information for Final Exam
November 26, 2013

General Information

The final exam will be given at the established time for 1:00 MWF classes – 11:30 am to 2:00 pm on Thursday, December 12. The exam will be written to be "doable" in about 1.5 hours if you are well prepared and work steadily. You will have the full 2.5 hour period to work, though, if you need that much time.

Miscellaneous Groundrules

No use of cell phones, pagers, I-pods, I-pads, or any other electronic devices beyond a calculator will be allowed during the exam. If your only calculator is on your phone, I can try to supply a few basic calculators for you to use, but *I cannot guarantee this. To be safe, obtain or borrow a calculator for use during the exam.* You will be asked to turn all other devices off and stow them in your backpack.

What Will Be Covered

The exam will cover the mathematical and historical material we have studied starting with the ancient Greeks. As you will see, though, one essay topic might ask you to put together some of this material with ideas we discussed earlier about Egyptian and Babylonian mathematics as well.

This means in particular you should be prepared for questions dealing with the following topics:

1. Greek mathematics

   a. Know the approximate historical periods and main contributions of the following figures: Euclid, Archimedes, Eratosthenes, Heron, Claudius Ptolemy, Theon, Proclus, where they came from in the Greek world (if known) and where they were active.
   b. Know the 5 Common Notions (Axioms) and 5 Postulates that appear at the beginning of Book I of Euclid's *Elements*.
   c. Know the statements and the proofs of Proposition 15 (vertical angles formed by the intersection of two lines are equal), Proposition 32 (the angle sum in a triangle is equal to two right angles) and Proposition 47 (the Pythagorean theorem)
   d. Possible question: What was the main result proved by Archimedes in his *Quadrature of the Parabola*? In general terms, without going into the technical details of his argument, how did he prove this? What later mathematical developments were Archimedes anticipating with this work?

2. Indian, Chinese, and Islamic mathematics

   a. Know in general terms the routes by which George Joseph proposes that Greek, Indian, and perhaps Chinese, mathematics were synthesized, developed and transmitted back to western Europe by the beginning of the Renaissance *Where and how* were the main connections between all of these different traditions formed?
   b. Know the approximate historical periods and main contributions of the following figures: Muhammad ibn Musa al-Khwarizmi, Thabit ibn Qurra, Omar Khayyam
   c. A possible question: What are *perfect numbers*? What are *amicable pairs of numbers*? Give examples of each and explain how they satisfy the definitions. How does the historical development of knowledge about these number-theoretical ideas tie together the Greeks, the Islamic mathematicians, and mathematics in the contemporary world?
   d. On the mathematical side, I might also ask you to show the diagram for the Chinese "go gou theorem" (with a general right triangle, sides a, b and hypotenuse c) and show how it provides an algebraic/geometric proof of the Pythagorean theorem
   e. Here is another geometric problem from a Chinese text by the author Yang Hui (about 1261 C.E.): There is a bamboo 10 chang (a unit of length) high. The bamboo is broken so that the upper end reaches the ground 3 chang from the foot of the bamboo. How high is the point where the break occurred?
   f. The following problem appears in a text by the Indian mathematician Mahavira (about 850 C.E.). Solve it using our modern notation and methods: Of a collection of mangoes, the king took 1/6, the queen 1/5 of the remainder, the chief princes $1/4, 1/3, 1/2$ of the successive remainders, and the youngest child took the remaining 3 mangoes. Oh you who are clever in miscellaneous problems on fractions, give the measure of that collection of mangoes.

3. From European mathematics, post 1500 C.E.

   a. Know the approximate historical periods and main contributions of the following figures: Leonardo Pisano ("Fibonacci"), Francois Viète, Nicolo Fontana ("Tartaglia"), Girolamo Cardano, Ludovico Ferrari, Filippo Brunelleschi, Leon Battista Alberti, Girard Desargues, Girolamo Saccheri, S.J., Nikolai Ivanovich Lobachevsky, Janos Bolyai
   b. Know the proof that shows that if ΔABC is a triangle, then under the alternate form (H) of Postulate 5, the sum of the angles in ΔABC is strictly less than two right angles, assuming as known our general facts about "Saccheri quadrilaterals."
   c. With the realization that non-Euclidean geometries are possible, what can we say about the relation between geometry as a mathematical subject and the geometry of physical space?

Format

This time, approximately 40% of the exam will consist of two or three short mathematical problems similar in format and content to the ones you have seen on the problem sets and in the list above. The remaining approximately 60% will be distributed, in a way to be determined, among some short answer questions combining the mathematics and the history, and an essay. This 60% will concentrate on aspects of the history and connections between various topics we discussed.

Essay

The essay question will be *one of the following*:

1. Albert Einstein, the well-known 20th century physicist wrote, provocatively: "As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain they do not refer to reality." Charles Darwin, the famous 19th century biologist wrote, rather humorously, "A mathematician is a blind man in a dark room looking for a black cat which isnt there." How does each of these statements relate to things we have discussed in this class, especially the geometry of Euclid's *Elements* and the roles of intuition and logic in mathematics? For example, do you think Darwin is alluding primarily to the *difficulty* of doing mathematics – finding and proving new theorems? Or is he saying something about how mathematics relates to the real world? Similarly, what does Einstein mean? If we are certain about a geometric theorem, is that saying something about reality? Or is the theorem about something else?

2. Here are two contrasting statements about the ultimate legacy of Greek mathematics:

    a. "The death of Archimedes by the hands of a Roman soldier is symbolical of a world-change of the first magnitude: the Greeks, with their love of abstract science, were superseded ... by the practical Romans. The Romans were a great race, but they were cursed with the sterility which waits upon practicality. They did not improve upon the knowledge of their forefathers, and all their advances were confined to the minor technical details of engineering. They were not dreamers enough to arrive at new points of view ... No Roman lost his life because he was absorbed in the contemplation of a mathematical diagram." (Alfred North Whitehead – 20th century philosopher)
    b. "There is no denying that the Greek approach to mathematics produced remarkable results, but it also hampered the subsequent development of the subject. ... The Greek preoccupation with geometry until the infiltration of the Mesopotamian and Egyptian influences in the later Hellenistic period was a serious constraint. Great minds such as Pythagoras, Euclid, and Apollonius spent much of their time creating what were essentially abstract idealized constructs; how they arrived at a conclusion was in some way more important than any practical significance." (George G. Joseph, *The Crest of the Peacock*)

    If you were to argue against Whitehead's statement, have we seen examples in this class where practicality was *not sterile*, where concern with applications actually enriched the history of mathematics and made further progress possible? For instance, what did the Greeks themselves say about the influence of earlier (and definitely more practical)

Babylonian and Egyptian mathematics on what they did? On the other hand, if you wanted to argue against Joseph's points, have we seen examples where the abstract idealized constructs of pure mathematics led to major advances? Why doesn't Joseph mention Archimedes? Does that weaken his point?