MATH 110 – Mathematics Through Time
Information for Midterm Exam
October 6, 2013

General Information

The first midterm exam will be given in class on Friday, October 25.

Miscellaneous Groundrules

No use of cell phones, pagers, I-pods, I-pads, or any other electronic devices beyond a calculator will be allowed during the exam. If your only calculator is on your phone, I can supply a few basic calculators for you to use. You will be asked to turn all other devices off and stow them in your backpack.

What Will Be Covered

The exam will cover the mathematical and historical material we have studied since the start of the semester, through the start of our discussion of the *Elements* of Euclid from class on October 7 and 9.

This means in particular:

1. Base 2,10,60 positional number systems.

2. Egyptian mathematics

   a. Know the approximate historical period represented by our primary Egyptian mathematical sources (the Rhind and Moscow mathematical papyri).
   b. Know the basic idea of their number system and the number symbols for 1's, 10's, 100's, 1000's (see BJB)
   c. "Egyptian multiplication and division" by repeated doubling
   d. Egyptian computations with unit fractions via tables of representations like the one from the Rhind papyrus on page ... of BJB).
   e. Be prepared for problems like the ones on Problem Set 1.

3. Old Babylonian mathematics

   a. Know the geographical area occupied by the Old Babylonian civilization and the approximate historical period represented by the Old Babylonian mathematical texts.
   b. Know their base-60 number system and number symbols and be able to read numbers in their format
   c. The role of addition, multiplication, "reciprocal," tables in Babylonian arithmetic

d. Know the different interpretations of the Babylonian problem texts like YBC 6967 given by Otto Neugebauer and Jens Høyrup.
e. Know what is on the "Plimpton 322" tablet and the different ways that Neugebauer and Eleanor Robson have explained what this tablet was.

4. Greek mathematics

a. Know the approximate historical periods of the following figures: Thales, Pythagoras, Plato, Euclid, where they came from in the Greek world (if known), and where they were active.
b. Know the 5 Common Notions (Axioms) and 5 Postulates that appear at the beginning of Book I of Euclid's *Elements*.
c. Know the statements and the proofs of Propositions 1 and 5 of Book I of the *Elements*. Be able to identify what Axioms, Postulates, and/or previously proved results are used in the proofs of these.

Format

Approximately 60% of the exam will consist of two or three short mathematical problems similar in format and content to the ones you have seen on the problem sets. The remaining approximately 40% will be distributed, in a way to be determined, among a few short answer questions and a short essay. This 40% will concentrate on aspects of the history.

Essay

The essay question will be *one of the following*:

1) "The distinguishing feature of Babylonian mathematics is its algebraic character." Of the historians we have mentioned, who would agree with this claim, and who would disagree? Explain using the the interpretations your historians would give for the YBC 6967 problem of solving the equation $x = 60/x + 7$.
2) How are the Egyptian and Old Babylonian ways of dealing with fractions different from each other, and different from what we do today? Which of these three systems has the capability of dealing with more fractions *in exact, finite terms* (that is, without resorting to infinite repeating expressions)? Explain.
3) Peter Rudman, author of a book about the extent to which the Pythagorean theorem was anticipated by the Old Babylonian problem texts states that "high school mathematics education today, with its emphasis on creating high scores on standardized tests, all too often neglects the derivations where mathematics is learned and emphasizes memorizing the equations that provide quick solutions in the standardized tests but that are then rapidly forgotten ... ." Is today's approach really all that different, though, from the Egyptian and Old Babylonian problem texts we have seen, where solutions of the problems posed are given as series of "do this, then do that" steps for

solving the problems? In your opinion, what is the most effective way that mathematics can be taught and learned? Does it matter what the specific goals of teaching mathematics are?

Practice Questions

The following are some practice questions indicating the types of mathematical questions I might ask on the midterm exam. Recall that these will account for about 60% of the total points. The remainder of the points will be distributed between short answer questions dealing with some of the historical background, and a short essay as described on the information sheet. There are answers for these questions posted on the course home page, but I think it will be best to work through complete solutions and then check your work – don't just look at the answer and say "yeah, that makes sense." You need to be able to do this for yourself with questions you are seeing for the first time.

Disclaimer

These practice problems show some of the range of topics that will be included and the approximate level of difficulty of the exam questions. The actual exam questions may look different and combine topics we have discussed in different ways.

Sample Questions

I.
A) Solve the equation $x^2 - 7x - 60 = 0$ by factoring the quadratic.
B) Solve the equation $x^2 - 7x - 60 = 0$ by means of the quadratic formula and show that the answers you obtain are the same as in part A.
C) Explain how the Old Babylonian problem text YBC 6967 can be understood as solving this quadratic equation.

II.
A) Express in base 10: $(101101)_2$.
B) Express in base 10: $(10, 23, 24)_{60}$.
C) Express in base 60: $(2691)_{10}$.

III. The following two questions are typical of the geometry problems from the Moscow mathematical papyrus. Solve them using modern methods.

A) The area of a rectangle is 48 and the width is 3/4 the length. What are the dimensions?
B) One leg of a right triangle is 5/2 times the length of the other. The area is 20. What are the dimensions?

IV. Compute "the Egyptian way:"
A) 56×125
B) $134 \div 24$

V. A Babylonian problem asks for the side of a square if the area of the square, minus the side is the base 60 number $(14, 30)_{60}$. The tablet says to do this to solve the problem (all numbers in base 60, of course!): "Take half of 1, which is $0; 30$, square that to get $0; 15$, add the $14, 30$ to get $14, 30; 15$. The last number is the square of $29; 30$. Now add the $0; 30$ to get 30, which is the side of the square."

A) Solve this problem using modern methods.
B) Is 30 the correct answer?
C) Explain how the Babylonian method of solution is essentially the same as using the quadratic formula(!)

VI. An integer n is *base-60 regular* if $n = 2^a 3^b 5^c$ for some $a, b, c \geq 0$ (that is, if n is divisible only by the primes occurring in the factorization of 60).

A) What are all the $n \leq 50$ that are base-60 regular?
B) Many Babylonian tables giving $n, 60/n$ are known and all of them contain only base-60 regular n's. Why do you suppose that is that true?

VII. I might also ask you to state and prove Proposition 1 or Proposition 5 from Book I of the *Elements* of Euclid.

Answers

I.

A) $x^2 - 7x - 60 = (x-12)(x+5)$, so $x = 12$ or $x = -5$.

B) By the quadratic formula

$$x = \frac{7 \pm \sqrt{49+280}}{2} = \frac{7 \pm \sqrt{289}}{2} = \frac{7 \pm 17}{2} = 12, -5.$$

C) The YBC 6967 problem is: Find a number x such that x exceeds $60/x$ by 7. This is equivalent to $x = 60/x + 7$, or $x^2 - 7x = 60 = 0$.

II. A) 45 B) 37404 C) $(44:51)_{60}$

III.

A) Call the length ℓ. Then $w = 3\ell/4$ and $A = 48 = \ell w = \frac{3\ell^2}{4}$. Solving for ℓ, $\ell = 8$ and $w = 6$.

B) Call the length of one leg ℓ. Then the other is $5\ell/2$ and the area is $20 = \frac{1}{2}\ell\frac{5\ell}{2}$. So $\ell^2 = 80/5 = 16$ and $\ell = 4$. The dimensions are $4, 10$.

IV.

A) By repeated doubling, we get $125, 250, 500, 1000, 2000, 4000$ (we can stop there since the next double would be 64×125, but $64 > 56$. Then since $56 = 32 + 16 + 8$,

$$56 \times 125 = 4000 + 2000 + 1000 = 7000$$

.

B) By repeated doubling, we get

$$1 \times 24 = 24$$
$$2 \times 24 = 48$$
$$4 \times 24 = 96$$

The next double is larger than 134 so we stop there. Then starting with $2/3 \times 24 = 16$, we get

$$2/3 \times 24 = 16$$
$$1/3 \times 24 = 8$$
$$1/6 \times 24 = 4$$
$$1/12 \times 24 = 2$$
$$1/24 \times 24 = 1$$

Since $134 = 96 + 24 + 8 + 4 + 2$, we have $134 \div 24 = 5 + 1/3 + 1/6 + 1/12$.

V.

A) Say the length of the side is x. Then the condition given says $x^2 - x = 870$, or $x^2 - x - 870 = 0$. By the quadratic formula we get

$$x = \frac{1 \pm \sqrt{1 + 3480}}{2} = 30, -28$$

Only the positive solution is relevant for the geometry, so $x = 30$.

B) Yes!

C) The Babylonian "directions" applied to any equation of the form $x^2 - px = q$ would say to compute

$$x = \sqrt{\left(\frac{p}{2}\right)^2 + q} + \frac{p}{2}$$

This is equivalent to the solution given by the quadratic formula(!)

VI.

A) $2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 27, 30, 32, 36, 40, 45, 48, 50$

B) Since those are the n for which $1/n$ has a *finite* base 60 expansion! (See Discussion 1 for one of those tables.)