MATH 110 – Mathematics Through Time
The Elements, Common Notions (Axioms) and Postulates
October 7, 2013

As we mentioned before, to avoid "infinite regress," *every deductive system must have unproved starting assumptions.* In *The Elements*, Euclid placed the following statements at the start of Book I to serve that purpose. The Common Notions (Axioms) are thought to represent valid ways of reasoning about any sort of quantitative information. The Postulates, on the other hand, are specific to the plane geometry that is the subject of Book I. The goal was to deduce *everything* from these basic unproved starting assertions.

Common Notions (Axioms)

1) Things that are equal to the same thing are equal to one another.
2) If equals be added to equals, the wholes are equal.
3) If equals be subtracted from equals, the remainders are equal.
4) Things that coincide with one another are equal to one another.
5) The whole is greater than the part.

Postulates (paraphrased)

1) (It is possible) to draw a straight line from any point to any point.
2) (It is possible) to produce any finite straight line continuously in a straight line.
3) (It is possible) to describe a circle with any center and distance.
4) All right angles are congruent to one another.
5) If a straight line falling on two straight lines makes the angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

Comments

The first three Postulates exactly describe the constructions possible with an (unmarked) straight edge and a "collapsing" compass. (That is, the compass can be used to describe circles, but not to measure and transfer distances.) The other two Postulates have a different character. Postulate 4 is, in effect, saying that the plane is "homogeneous" (a right angle at any one point is congruent to a right angle anywhere else). The fifth postulate is both more complicated and less obvious than the other ones. In high school, you may have seen an alternate statement called the Playfair Postulate which is equivalent:

5') Given a line and a point not on the line, there is exactly one line parallel to the given line passing through the point.