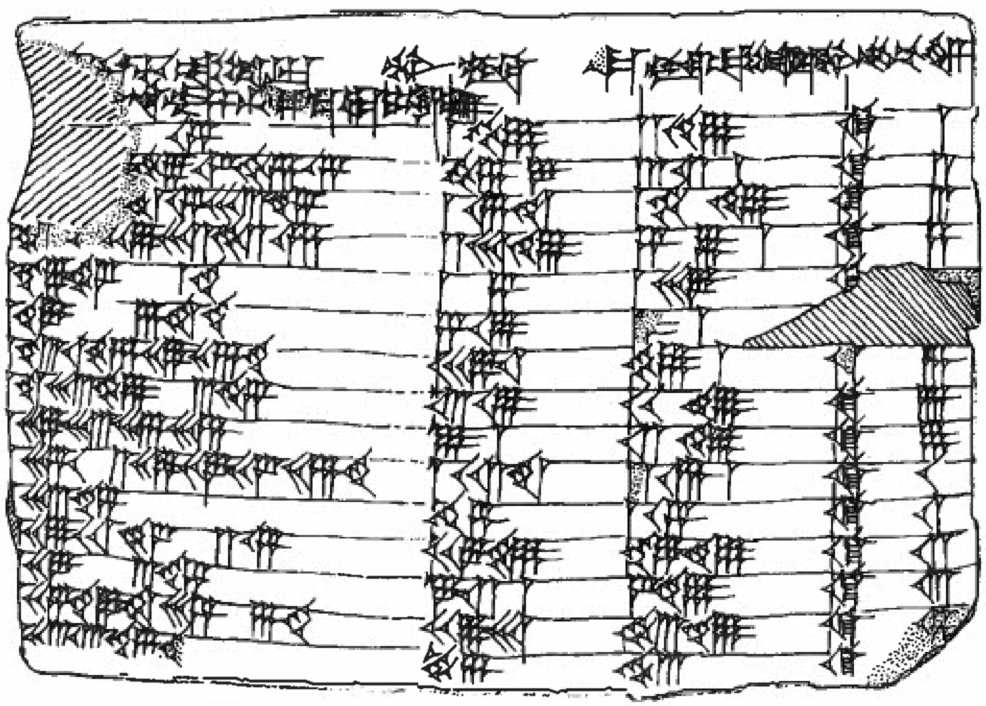
MATH 110 – Mathematics Through Time
Discussion 2 – "Plimpton 322"
September 20, 2013

Background

The picture below is a modern drawing (made by the historian Eleanor Robson) of the Old Babylonian mathematical tablet known as "Plimpton 322" (catalog number 322 in the G. A. Plimpton collection at Columbia University in New York).



There is a photograph of the original tablet on our class homepage and also in the Joseph and Bunt/Jones/Bedient books. As you will see, the reflection of light off the surface of the clay and the depth of the stylus marks make that somewhat difficult to read. Unfortunately, the tablet also has some damaged regions where chips of the original surface have been lost (indicated by the hatched areas here). So I think you will find that this version is much easier to interpret and work with.

You may also enjoy looking at the color photograph of Plimpton 322 on the web site of the "Before Pythagoras: The Culture of Old Babylonian Mathematics" exhibition from 2010 at the Institute for the Study of the Ancient World in New York:

`http://isaw.nyu.edu//exhibitions/before-pythagoras/`

That exhibit included the tablets YBC 7289, Plimpton 322, and a number of others.

The top two rows of cuneiform on the tablet contain a somewhat cryptic text whose exact meaning is not completely understood, and that we will not consider for now. The rest of the tablet consists of 5 columns cuneiform, mostly number symbols. Let's call those columns A,B,C,D,E, starting *from the right.*

Questions

A) Look at the column on the right hand edge (column A). What are those numbers and why do you suppose that column was included?

Column B contains the same cuneiform symbol repeated in each row. This is a word related to the text at the top. We will ignore this. The next three columns are entirely composed of numbers. They will be our focus in today's work (especially columns C and D). For instance you should see that the numbers in columns C, D in row 1 are

$$1;59 = (119)_{10} \qquad \text{and} \qquad 2;49 = (169)_{10}$$

Note that if we subtract the squares of these numbers, the difference

$$169^2 - 119^2 = 14400 = 120^2$$

is another perfect square(!) In other words $a = 119$, $b = 120$ and $c = 169$ form what modern mathematicians call a *Pythagorean triple* – a triple of integers (a, b, c) satisfying the equation $a^2 + b^2 = c^2$.

B) Identify and translate the rest of the numbers in columns C and D. Write them in our convention for base 60 numbers, and then as base 10 numbers. If there is some ambiguity or the surface of the tablet is damaged in one of those entries, you can either try to guess to fill that entry in or just indicate that there was a problem with that entry.

C) What happens if you take the number on a row in column C, square it, and subtract the square of the number on the same row in column D. What is true about the numbers you get? (There should be some "exceptions" to a general pattern you notice in most rows. In fact, it appears that there are some *errors* in the table!)

D) (Harder!) What about column E (the left most column). Unfortunately note that there is a big "chunk" missing out of the first several rows on the left. However, the lower portion of that column is better. Look at row 11 carefully, for instance. From the rows above, it appears that there should always be a 1 at the start, so this number is

$$1;33;45$$

How does that relate to the entries c, d on that row in columns C and D and the number $c^2 - d^2$? Try a few of the other rows that followed the pattern from part C and see if it always works this way.