Background

Today, we want to look at one important source of information about early mathematics in India, the *Sulba Sutras.* There are several of these texts known with variations of different sorts. The main idea of all of them was the use of geometric relationships in the construction of fire altars for the ancient Vedic religion. The following information is given in one of these texts for the construction of a particular great altar.

AX = XD = 12 *padas*
BY = YC = 15 *padas*
XP = 5 *padas*
PR = 23 *padas*
RQ = 7 *padas*
QY = 1 *pada*
XY = 36 *padas*

FIGURE 8.3: The layout of the *Mahavedi* (Great Altar)
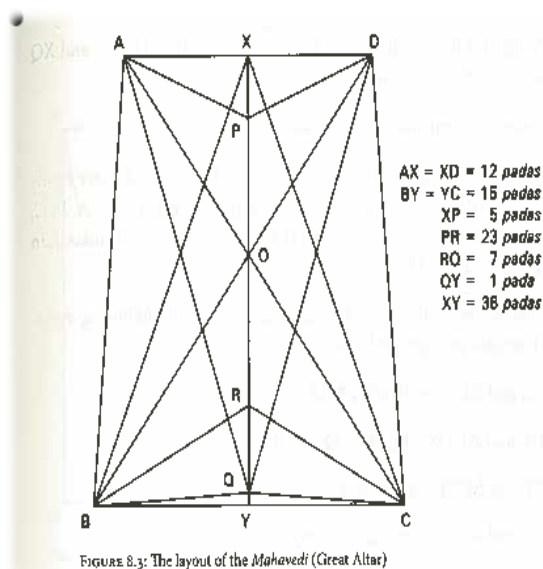
Our goal is to discover some of the numerical relationships occurring here and to interpret them in the light of one of the mathematical statements found in these texts.

Questions

(A) Given that all of the triangles with only horizontal and slanted sides in the figure are isosceles with symmetry across the line containing X, Y, show, using facts from Book I of the *Elements* that that XY meets AD and BC at right angles at the points P and Y. Hint: Use triangle congruences.

(B) Now, given the information in the legend at the right of the figure, determine

   1. The lengths $AP = PD$ (Hint: what is true about the triangles $\triangle APX$ and $\triangle DPX$?)

   2. The lengths $AY = YD$ (note: it's not entirely legible in the scanned image, but $XY = 36$)

   3. The lengths $CX = XB$

4. The lengths $BR = RC$

5. The lengths $AQ = QD$

You should have noticed some intriguing patterns. What is going on in this figure? What is common to all of these pairs of triangles?

(C) In the text of all of the *Sulba Sutras* is the following statement: *A rope stretched along the length of the diagonal of a rectangle makes an area which the vertical and horizontal sides make together.* How would we say this? What is this result?

(D) (Short Essay/Thought Question) Recall the way Høyrup's cut and paste interpretation of the steps on YBC 6967 (about 1800 BCE) is very close to one way of deriving the Pythagorean theorem. That way is essentially the same as the the figure for the *gou* theorem from the Chinese *Zhou Bi Suan Jing*, which might date to 500 BCE or earlier. All of the examples in (B) and the statement from (C) are from Indian texts thought to date from about 800 BCE. Proclus, writing about Euclid's proof of the theorem in Book I of the *Elements* about 480 CE said, "If we listen to those who like to record antiquities, we shall find them attributing the theorem to Pythagoras and saying that he sacrificed an ox on its discovery. For my [i.e. Proclus's] part, though I marvel at those who first noted the truth of this theorem, I admire more the author of the *Elements*, not only for the very lucid proof by which he made it fast [i.e. the proof of Proposition 47 in Book I], but also because he showed an even more general statement in Book VI." Given all of this, why do you suppose we still call this result the *Pythagorean theorem*? Do you admire more those who discovered the theorem or Euclid for finding such a systematic and beautiful proof?