Mathematics through time

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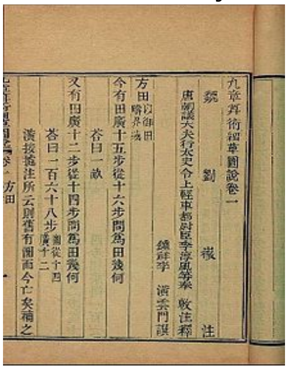
An introduction to ancient Chinese mathematics briefly covering features of Jiu Zhang Suan Shu, Chinese Remainder Theorem, and Hsuan-thu

Traditional Chinese mathematics was developed in its cultural context characterized as authoritarian. Ancient Chinese government established strict central power and demanded political obedience that stifled creativity and curiosity necessary to discover the truth of the nature. For more than one thousand years, research and education of mathematics were under the control of the Chinese imperial government. The traditional Chinese government did not like the idea of establishing domestic Academies like the Platonic Academy in 387 BC in ~~Athens~~, to openly study and research mathematics. Most of the traditional Chinese mathematicians were government officials specializing in researching astronomy, and they had their exclusive access to profound mathematic records. Traditional Chinese mathematics was developed as the collection of intelligence of geniuses who discovered the truth of nature occasionally through own intellectual sparks. After Yuan Dynasty (1271-1368), because the government appraised only the value of studying “Eight - Part Essays” – a Chinese literary form and genre, mathematics was made to be an inferior form of knowledge which was eventually annihilated from its imperial examination system as a test subject, thus the development of mathematics was stagnant.

The Chinese cultural background highlights the utilitarian purposes of Chinese mathematics. Due to lack of creativity and incentives, ancient Chinese had developed mathematics to serve the need to measure and communicate about time, quantity, and distance in real world applications. For example, Jiu Zhang Suan Shu, also called Nine Chapters on the Mathematical Art, was the most famous book which shows the traditional Chinese approach of finding general methods of solving problems of a particular kind. In this book, the Chinese, for the first time in world history, demonstrated operation with fractions, as well as methods of adding and subtracting negative numbers. What particularly interesting was that ancient Chinese invented these advanced mathematic calculations to train people to become mathematically competent administrators. Jiu Zhang Suan Shu contains 246 mathematic problems, covering practical areas including measurement of weights, trade, taxation, commuting, payment of wages, and so on.



九章算術細草圖說卷一
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錢塘李潢雲門譔
方田以御田疇界域
今有田廣十五步從十六步問為田幾何
荅曰一畝
又有田廣十二步從十四步問為田幾何
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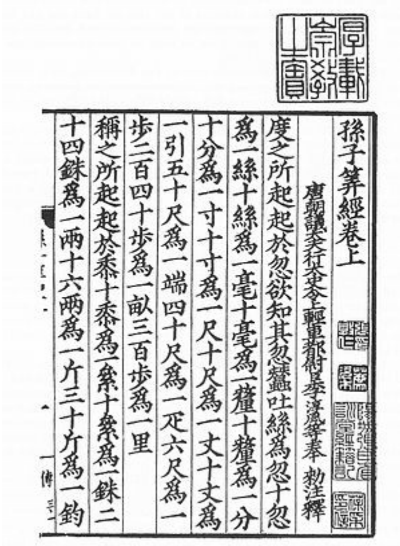
This utilitarian nature distinguishes ancient Chinese mathematics from ancient Greek mathematics. Both Euclid and Plato demonstrated to us that the beauty of ancient Greek mathematics lies on the systematic reasoning according to some generally accepted interpretations of mathematical principles. Plato was especially concerned about the process of reaching a conclusion through a series of logical reasoning. In contract, it seemed like for ancient Chinese mathematics, reaching an approximate numerical result was of top priority. For example, ancient Chinese have found the formulas for calculating areas of geometric shapes including rectangle, circle, and triangle and so on. But for simplicity, in Jiu Zhang Suan Shu, π was only rounded up to 3 in solving problems. The reason why ancient Chinese did so was that the round-up was sufficiently accurate for real job applications in any aspects like architecture, agriculture, trading and so on.

Despite simplifying complicated mathematical problems for solving daily life issues, traditional Chinese mathematics had inventions that, in their time, were leading the world for its profound computing techniques. At Wei-Jin, Southern & Northern Dynasties periods, social unrest and famine were rampant during the time of war. Minds originally bound by social oppression were freed and Chinese mathematics experienced substantial development theoretically and flourished vigorously. Among the most famous theorems invented, the Chinese Remainder Theorem was an important generalization in arithmetic theory that amazed the world.

Chinese Remainder Theorem was first published in the book ~~Sunzi's Mathematical Classic~~. The original problem leading to the invention of this great theory can be interpreted in translation as: a number divided by 3 has a remainder 2, divided by 5 has remainder 3, divided by 7 has reminder 2. Find the smallest natural number suitable for these conditions. In mathematical form, the problem can be presented as: N = 2 (mode 3) = 3 (mode 5) = 2 (mode 7). The solution is: N = 23. To draw a generalized statement to solve this particular kind of problem, Sun Tzu, the author of the book, gave the following proof: Assume a, b, c, d are positive integers, a number divided by 3 has a remainder a, divided by 5 has remainder b, divided by 7 has reminder c. Then from the conditions given we can conclude that $m = 70a + 21b + 15c – 105d$ (Sun Tzu did complicated calculations to reach this point). After that, Sun Tzu proved that $m = 70a + 21b + 15c – 105d$ can be written as $m = (69a + 21b + 15c – 105d) + c$. And because $3|(69a + 21b + 15c – 105d)$ and $0 \leq a \leq 2$, m divided by 3 must have a remainder a. Similarly, m divided by 5 has a remainder of b, and m divided by 7 has a remainder c. And since [3, 5, 7] = 105, m minus integer multiples of 105 will give the smallest natural number for the problem. Later in history, Chinese Remainder Theorem has provided a foundation for building Number theory, polynomial theory, assignment theory, and cryptography and so on.

A page of ~~Sunzi's Mathematical Classic~~:



孫子算經卷上

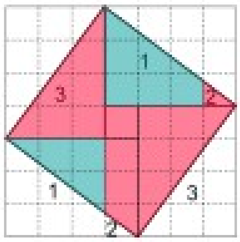
唐朝議大夫行太史令上輕車都尉臣李淳風等奉 勑注釋

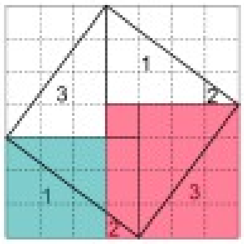
度之所起起於忽欲知其忽蠶吐絲爲忽十忽爲一絲十絲爲一毫十毫爲一氂十氂爲一分十分爲一寸十寸爲一尺十尺爲一丈十丈爲一引五十尺爲一端四十尺爲一疋六尺爲一步二百四十步爲一畝三百步爲一里

稱之所起起於黍十黍爲一絫十絫爲一銖二十四銖爲一兩十六兩爲一斤三十斤爲一鈞

Traditional Chinese government was a typical autocratic monarchy which had its own limitations and strains on the development of Chinese mathematics. Traditional Chinese mathematical thoughts experienced a transition to understand Euclid’s axiomatic deductive system through learning from Western ways of dealing with mathematics. Near the end of the 16th century, Western missionaries began to come to China, with the purpose of spreading Christianity and Western sciences cross cultures. In 1607, Matteo Ricci, as a missionary, not only passed one of the most brilliant exotic mathematic thoughts to China through translating Euclid’s Elements into Chinese, but made significant contributions to replenish, refresh and promote Chinese mathematical thinking. Traditional Chinese mathematics focused on mechanistic rules that propel algorithmic thinking. Euclid’s Elements, for the first time in Chinese history, shocked the Chinese through emphasizing the importance of a continuity of space and time.

Euclidean-style of geometry is a prime example of the use of deductive reasoning. Compared to Euclid, traditional Chinese was particularly good at boiling down geometric problems to algebraic equations, and then use algorithm to solve the probelm. In the book Zhou Bi Suan Jing, Zhao Shuang gave the first recorded proof of the Pythagorean Theorem through constructing hsuan-thu. Zhao Shuang proved Pythagorean Theorem through the investigation of cut-paste algorithm in overlapping grid. In hsuan-thu, 4 identical right triangles that have sides of 3, 4, and 5 respectively form a square. The hypotenuse, which is the longest side of each right triangle, becomes each side of the square, as depicted in the graph on the left. Then, Zhao Shuang proved that the area of the whole square equals to 5^2, which has the same area as the sum of areas of two squares with sides of 3 and 4 respectively -- $3^2 + 4^2$. Therefore, the general formula for the area of any right triangle that has sides a, b, and c would be $c^2 = a^2 + b^2$, in which c is the hypotenuse that is the longest of the three sides. Zhao Shuang's proof of Pythagorean Theory is strictly rational but easily understandable. The design of the logo of 24th International Congress of Mathematicians (ICM) held in China was inspired by the pattern of Zhao Shuang's hsuan-thu. Zhao Shuang's hsuan-thu was selected to represent the height of ancient Chinese mathematics, and the crucial role it played in the development of mathematics.





In sum, Chinese mathematics is based on the creation of a variety of algorithmic equations. From linear equations to polynomial equations, the Chinese created a series of advanced algorithms (what the Chinese calls “Shu”) to solve corresponding types of algebraic equations, thus solving both scientific and practical problems. Chinese Remainder Theorem and Zhao Shuang’s hsuan-thu all illustrate the Chinese mechanistic approach that uses algorithmic equations to find general equation of solving problems of a particular kind. In addition, the pre-existing cultural context and imperial system had fundamentally contributed to the development of Chinese. All in all, Chinese mathematics, with its extremely rich heritage, not only has propelled scientific development throughout history, but will guide people from all over the world to embrace future mathematical learning and discovery.

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