MONT 101N – Analyzing Environmental Data
Some Sample Final Exam Questions
April 27, 2018

1. Briefly explain the following chance and statistical concepts.
   (a) Central Limit Theorem
   (b) 95% condence interval
   (c) t-distribution
   (d) standard normal distribution
   (e) Null hypothesis of a test
   (f) p-value of a test of hypotheses

2. Short Answer:
   (a) (Hypothetical) A college has an elective quantitative reasoning course for first-year students. Each year approximately one fifth of the first-year students elect to take this course. The college does a study of the grades of first-year students. The study shows that after the first- year of college, the students who elect to take this math course have an average GPA for their other three courses that is .1 units higher than the average GPA for the other students for all their courses. Based on this data, the Mathematics department argues that this course raises student GPAs and that every student should be required to take it. Does the Mathematics Department have a good argument or is it possibly flawed? If it is a good argument, explain why, if not, how would you correct the study?
   (b) Why are we justified in using a normal curve to make estimates about sample means with sample sizes $N > 30$ even if the population the individual Y_i are sampled from is not normally distributed?
   (c) In major national polls, which guide politicians and political candidates in their decision making and play a prominent role in the national media, the sample size is usually 1000 or so. Why are samples of this size used? In particular, what are the benefits of using a sample size of 1000 as opposed to smaller sample sizes or larger sample sizes?
   (d) If you want to get polling results with comparable accuracy, do you need a bigger sample size if the polling is done in California than you do if the polling is done in Rhode Island? Explain.
   (e) True or False and explain: If the p-value of a test of significance works out to $p = .05$ then we can say that the null hypothesis was false.
   (f) True or False and explain: Increasing the sample size N will *always* decrease the width of a 95% confidence interval for an average. (Assume $N > 30$ to start with.)

3. Let Z represent a standard normal random variable and X represent a normal random variable with $\mu = 4.5$ and $\sigma = 1.3$.

(a) Find $P(1.24 < Z < 2.0)$

(b) Find $P(-0.4 < Z < 1.33)$

(c) Find $P(X > 5)$

(d) Find $P(4 < X < 5.4)$

(e) Suppose that values of X are sampled $n = 12$ times independently. What is the probability that 4 out of the 12 values satisfy $4 < X < 5.4$.

(f) Now suppose you have a (very) large urn containing 10000 balls, 6069 of them blue and 3031 of them red. Suppose you picked 12 of them at random *without replacement*. What is the probability that you get exactly 4 blue balls?

4. A Gallup poll released on April 3, 2017 surveyed a random sample of 706 adults nationwide. It reported "that 59% of Americans say the environment should be prioritized over energy production."

   (a) Determine a 95% confidence interval for the percent of Americans who actually feel this way. Use the "conservative" method of computing the standard error.

   (b) True or False and explain: There is a 95% chance that the actual percent of Americans who feel this way is in the interval you computed in part b.

5. Before a person gives blood, the Red Cross requires that the hemoglobin in their blood measures 12.0 or higher on an electronic scan. There is typically some error in the measurements. (The following is hypothetical.) Suppose that such a device is being calibrated by five readings on a standardized sample known to have a hemoglobin content of 12.0. The five readings are 11.5, 11.9, 12.0, 12.1, 12.1 for an average of 11.92. Based on these readings, we want to do a hypothesis test to determine whether the device is calibrated correctly or not.

   (a) Formulate a null and alternative hypothesis for the difference between 12.0 and 11.92.

   (b) What test should you use? Why?

   (c) Carry out your test. What do you get for a test statistic and observed significance level?

   (d) Is the device calibrated correctly, or is it biased?

6. (This problem is adapted from "Heart rate in yoga asana practice: A comparison of styles," Journal of Bodywork and Movement Therapies Volume 11, Issue 1, January 2007, Pages 91-95.) Yoga is often recommended for stress relief, yet some of the more fitness-oriented styles of yoga can be vigorous forms of exercise. The purpose of this study was to investigate differences in heart rate during the physical practice of yoga postures, breathing exercises, and relaxation. The study led groups of participants through three different styles of yoga: astanga yoga, hatha yoga, and "gentle" yoga. Participants wore heart rate monitors during the sessions and their heart rates were monitored repeatedly throughout the sessions. Assume there were three independent groups of 50 participants each in the study. Each group was put through a practice session with one style of yoga. The average and standard deviation

for the heart rates (in beats per minute) of the participants for each style of yoga are given below:

Yoga style	Average heart rate (bpm)	SD
astanga	95	12.84
hatha	80	9.32
"gentle"	74	7.41

The researchers then applied two-sample z-tests to each pair of styles and concluded that there may be different fitness benefits for different styles of yoga practice.

(a) Apply a two-sample z-test to the astanga and hatha yoga data. What is your p-value?

(b) Apply a two-sample z-test to the hatha and gentle yoga data. What is this p-value?

(c) Were the researchers' conclusions justified? (Note – the higher the average heart rate attained, the higher the fitness benefit.)