MONT 100N – Modeling the Environment
Chapter 7 Project – Using Natural Resources Wisely.
November 15 and 17, 2017

Environmental scientists often try to estimate populations of plant or animal species and understand to what extent they can be used as resources by humans without being depleted. For instance, populations of wild fish and other marine creatures around the world have been a major source of food for humans for many years. Yet there is evidence that many of them have been overfished and there is fear some of them may be headed toward extinction. For example, following 500 years of fishing, by the summer of 1992, the biomass of northern cod observed in Atlantic waters off the coast of Newfoundland had fallen to an estimated 1% of its previous levels. As a result, the federal government of Canada declared a moratorium on cod fishing, hoping to give the cod populations time to recover. The economic and social impacts on the human population of Newfoundland were severe, since many of the people in the area derived their entire livelihood from cod fishing and others depended on cod fishermen as customers for their businesses. An estimated $35,000$ people lost their jobs as and the whole society of Newfoundland has not really recovered to this day. As of around 2010, there were some encouraging signs that the cod fishery might be recovering, but the effects of other factors such as changes in ocean temperature and loss of populations of the food species that cod eat have kept the ultimate fate of the north Atlantic cod fishery uncertain. Similar decreases have been observed in New England cod populations more recently and the U.S. government has instituted more and more stringent fishing limits to try to avert collapses here as well. The fishing industries of Maine and Massachusetts are under similar pressures.

In this project, you will study various models of a fishery including effects from fishing by humans. Let $P(n)$ represent the total mass of mature Pacific halibut in units of 10^6 kg. We will model the wild halibut biomass *without any fishing* by the following logistic difference equation

$$P(n+1) = (1.71) \cdot P(n) - (.00875) \cdot (P(n))^2. \tag{1}$$

Here $r = .71$ and $r/M = .00875$.

Questions

(A) *"Greedy" harvesting.* Suppose the halibut stock started out at 95% percent of the carrying capacity according to the model (1). But in one massive fishing effort, the halibut biomass is reduced all the way down to 1×10^6 kg (say all within one year). If no further fishing is allowed until stocks recover to 95% of the carrying capacity, how long will that take, according to the model? Estimate by computing a solution of the difference equation (1). What would be the *average fish amount taken per year* if this process of massive fishing followed by fallow time to allow recovery to 95% of the carrying capacity was done repeatedly over a long period?

(B) *Constant harvesting.* One way to make use of a resource like the halibut fishery that is less drastic than the "greedy" approach in (A) is to take some constant amount of fish every year.

(1) Suppose that everything remains as in (1) above, but some constant amount h (in 10^6 kg)

of halibut biomass is removed each year via fishing.[1] What modified difference equation models this situation? (Think about the derivation of the logistic equation

$$Q(n+1) = (1+r)Q(n) - \frac{r(Q(n))^2}{M} \tag{2}$$

and take the fishing amount h into account.)

(2) Investigate the solutions of your constant harvesting difference equation from part (1) if the fishing term is each of these values: $h = 5, 10, 14, 20$, one at a time. Choose enough different $P(0)$ values for each so that you think you see the whole picture and then describe what is happening in words. In particular, for each h how many different equilibrium solutions are there? Where are they located? How do they change as h increases? Are they stable or unstable?

(3) By rewriting your difference equation from part (1) in the form

$$P(n+1) - P(n) = \cdots ,$$

what is the *maximum value* of h for which the equation still has a stable equilibrium? (This question can be answered by means of algebra alone if you think about it the right way!)

(4) What should it mean to say that a fishing level h is *sustainable*? What is the maximum sustainable constant fishing level? Does the answer depend on what the initial value $P(0)$ at the start of the fishing intervention is?

(5) What would be the *average fish amount taken per year* if constant harvesting at the maximum sustainable level is done repeatedly over a long period?

(C) *Proportional harvesting.* Instead of taking a constant amount of fish, we could also take a constant proportion of whatever fish biomass is present.

(1) Next, suppose that everything remains as in (1) above, but instead of a constant amount, suppose that a constant *proportion* p of the halibut biomass (whatever it is) is removed each year via fishing.[2] What modified difference equation models this situation? (Think about the derivation of Equation (2) and take the proportion removed by fishing into account.)

(2) Investigate the solutions of your constant harvesting difference equation from part (1) if the fishing term is each of these values: $p = 0.1, 0.3, 0.5, 0.8$, one at a time. Choose enough different $P(0)$ values so that you think you see the whole picture and then describe what is happening in words. In particular, for each value of p how many different equilibrium solutions are there? Where are they located? How do they change as h increases? Are they stable or unstable?

[1] Think h means a "harvesting level," hence the notation.
[2] Think $0 < p < 1$ with $p = 1$ meaning all of the fish are removed.

(3) By rewriting your difference equation from part (1) in the form

$$P(n+1) - P(n) = \cdots,$$

what is the p for which the halibut population starts to "crash" for all $P(0)$? (This question can be answered by means of algebra alone if you think about it the right way. And the answer should make biological sense too!)

(4) What should it mean to say that a fishing proportion p is *sustainable*? What values of p are sustainable? Does the answer depend on what the initial value $P(0)$ at the start of the fishing intervention is?

(5) What would be the *average fish amount taken per year* if proportional harvesting at the level $p = .3$ is done repeatedly over a long period?

(D) Compare the strategies in parts (A), (B), (C) from the point of view of their effect on the halibut fishery and the average amounts taken per year. If you were going to recommend one, which would it be? Explain how you are making your determination.