MONT 100N – Modeling the Environment
Chapter 1 Project
September 15, 2017

Background

The ice sheet covering the island of Greenland is currently one of the major concentrations of frozen fresh water situated over a land mass on the Earth. It is estimated to contain about $\frac{1}{9}$ of the total freshwater ice on the planet. Because this water is presently on land, if it should melt, that water would add to the depth of the oceans. This is different from the situation for the Arctic sea ice, or much of the ice in the shelves surrounding Antarctica, which rest on sea water. It is thought that melting in those cases would not appreciably change sea levels because that ice is floating on and supported by water already.[1] There are important freshwater ice sheets covering the land mass of Antarctica as well as these sea ice formations, though, and their water *would* add to the depth of the oceans.

As we will see, there is a large volume of water contained in the Greenland ice sheet. For this reason, its fate is of more than passing interest for all humans–many of our major cities and settled coastal areas occur in regions that are close enough to the current sea level that any significant increase will cause major disruptions. For instance, the *highest* (natural) elevation of any point on the island of Manhattan in New York City is only 81 meters ($\doteq$ 265 feet) above sea level and most of the island lies much lower than that. The effects of the relatively small increase in average sea level that have already occurred were evident, for instance, in the flooding of Manhattan that happened during "super-storm Sandy" in October, 2012.

The news on this front is not encouraging because major melting has been observed on Greenland in recent summers. This is thought to be a result of a combination of higher air temperatures and decreases in the reflectivity of the ice due to deposition of soot particles from sources on other continents (this increases the amount of energy absorbed from solar radiation during the summer months and increases the rate of melting).

Refer to the map of Greenland showing the depth of the ice sheet covering most of its land area in Figure 1. We want to use the information here to estimate the volume of the water contained in this ice sheet and understand the possible (now maybe even probable!) effects if it melts completely.

Our methods will yield rough estimates or approximations of the ice volume. They will be based on the following simplifying assumptions:

- Even though it appears as a large area on the familiar Mercator projection maps you may have seen, that map projection distorts areas of regions near the poles and makes them look much larger than they actually are. Greenland does not make up a very large a portion of the surface area of the (roughly spherical) Earth. As a result we will not lose too much if we simply estimate areas as though they corresponded to areas on the flat map (i.e. without trying to take the curvature of the Earth into account). The distance scale marked in the

[1]This is good since as of July 12, 2017 the Larsen C ice shelf, with an area of about 44,200 square kilometers–about the same as the land area of the U.S. state of Delaware–has broken off from the Antarctic coast and become what is probably the largest iceberg ever observed by humans.

Figure 1: The Greenland Ice Sheet, as of about 2013, source `https://en.wikipedia.org/wiki/Greenland_ice_sheet`

legend of the map can be used, together with a ruler, to approximate linear dimensions of regions.

- The ice depths (in meters) of the various portions of the ice sheet are encoded in the map by the 6 different colors. For the purposes of this exercise, let's take the ice depth in a region to be *constant* at the *lower limit* of the range of depths shown in the legend of the map. For example, this will mean that all points colored with the darkest shade of blue will have ice depths equal to 3000 m (even though the actual depths can be larger). Note that this will produce estimates on the small side of the actual depth.

Questions

The chapter project will involve investigating the following questions and writing up your results as directed below.

(A) Estimate the areas of each of the five different ice sheet regions identified by the colors. Each group will get transparencies I traced by hand showing the *outside boundaries* of the regions in the map where the ice depth is at least 10 meters (solid black), at least 1000 meters (dotted black), at least 1500 meters (green), at least 2000 meters (blue) and at least 2500 meters (red). Explain how you are doing this in a clearly-written paragraph. Note: There are *many ways* to do this in a reasonable fashion and there is not a single correct answer! One suggestion: Use the rectangular grid marked on the second transparencies that your group will receive. *The grid squares marked are* $200km$ *on a side according to the scale of the map.* You don't need to get super-detailed or picky, but be as accurate as is reasonably possible.

(B) Multiply each of your area estimates by the depth estimate to get a volume estimate. Add the ice volume estimates to get a total volume and express in units of *cubic kilometers*. (As a "reality check" for your method, the total volume of the Greenland ice sheet is often estimated to be about $3,000,000$ cubic kilometers. How close did you come to that?) Recall that the Greenland ice sheet accounts for about $\frac{1}{9}$ of the total freshwater ice on Earth. What is your estimate of the total volume of freshwater ice on Earth?

(C) Now imagine that an amount of water equal to your total volume estimate is added to the oceans all at once. How much would sea levels rise as a result? One way to estimate that is to use the same idea as in part (3) of Example 1.5 in the text. What is the total surface area of the oceans on the Earth? (You should look this up online if you don't know what proportion of the Earth's surface area is accounted for by the oceans. If you find different estimates, how will you choose which one to use? Explain your thinking.) If the water from the melted Greenland ice sheet was spread evenly over that area, how deep would it be, in meters? Would the actual change in sea level be less than or greater than this estimated height? Explain.[2]

[2]A technical note: In case you are worried about the fact that we are ignoring the spherical shape of the Earth, this method is actually sufficient (i.e. accurate enough) for our purposes because of the fact that the depth of the

(D) Find an elevation contour map of Manhattan. Use that information to estimate what portions of that island would be under water if all the freshwater ice sheets melted.

Write up your solutions for these questions as a project report. Include all of your calculations, the version of the map you used to estimate the areas of the different regions of the ice sheet, and your answers to all the "explain" portions of the questions above.

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water would be much smaller than the radius of the Earth. Here's one way to think about it: If there is no land area, then adding the water from the melted ice sheet is equivalent in mathematical terms to changing the radius of the spherical Earth from r to $r + \Delta r$ (with Δr representing the depth of the new water, much smaller than r itself). The volume of the added water is the difference between the volume of the larger sphere of radius $r + \Delta r$ and the volume of the original sphere of radius r:

$$\frac{4\pi(r + \Delta r)^3}{3} - \frac{4\pi r^3}{3} = 4\pi r^2 \times \Delta r + 4\pi r \times (\Delta r)^2 + \frac{4\pi(\Delta r)^3}{3}.$$

Since we assume Δr is much smaller than r, then the last two terms on the right are negligible in size compared to the first term and we obtain an estimate

$$\text{volume of added water} \doteq 4\pi r^2 \times \Delta r = \text{surface area of sphere} \times \Delta r.$$

The change in sea level is then approximated by

$$\Delta r \doteq \frac{\text{volume of added water}}{\text{surface area of sphere}}.$$

The same idea works even if the water covers only a portion of the surface area of the sphere.