Mathematics 243 – Mathematical Structures
Mathematical Statements
August 30, 2017

Background

In mathematics, a *statement* is a sentence that is unambiguously either *true* or *false*.

- For instance "17 is an odd number" *is a statement* because the concept of even and odd numbers is defined, and we have an unambiguous way to test every whole number to see whether it is even or odd. That statement is true (because odd numbers are those of the form $2k + 1$ for some k and $17 = 2 \cdot 8 + 1$).
- On the other hand, the equation that we think of as one form of the Pythagorean theorem: "$a^2 + b^2 = c^2$" is not, by itself, a statement. Why not? Well, first, note that whether the equation is true depends on what a, b, c are(!) That is enough to disqualify this as a statement.

Discussion Questions

(A) What would you have to add to the equation "$a^2 + b^2 = c^2$" to produce a true statement (the statement of the Pythagorean theorem)?

(B) Which of the following are mathematical statements (in the strict sense described above)? Which of those that are statements are true and which are false? Explain how you can tell. For those that are not statements, can you add something to produce a statement?

   (1) The number 43 is prime.

   (2) The function $f(x) = \sin(e^x)$ has an interesting graph.

   (3) Every triangle is isosceles.

   (4) If $x = \pi/3$, then $\cos(x) = 1/2$.

   (5) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

   (6) $x^3 + 5x + 6 = 8$ for some real number x between $x = 0$ and $x = 1$. (This is a complete sentence, in a way, but where's the *verb*?)