Math 131 - section 01 - Precalculus Diagnostic Quiz Answers
August 31, 2007

Circle the correct answer. **Show your work.** Please turn over for problems 4,5,6.

1. Simplify: $\sqrt{15} \left( \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}} \right)$

   A. $2\sqrt{\frac{15}{8}}$  B. $\frac{\sqrt{15}}{\sqrt{3} + \sqrt{5}}$  C. 1  D. $\sqrt{3} + \sqrt{5}$  E. $2\sqrt{3}$.

   - **Solutions:** The answer is D. There are several ways to do this, but all rely on the fact that $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ for all positive real $a, b$.
   
   - Method 1: Multiply through:
     
     \[
     \sqrt{15} \left( \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}} \right) = \frac{\sqrt{15}}{\sqrt{3}} + \frac{\sqrt{15}}{\sqrt{5}}
     \]
     
     \[
     = \frac{\sqrt{5}\sqrt{3}}{\sqrt{3}} + \frac{\sqrt{5}\sqrt{3}}{\sqrt{5}}
     \]
     
     \[
     = \sqrt{5} + \sqrt{3}
     \]

   - Method 2: Put the terms on the inside the parentheses over a common denominator:

     \[
     \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}} = \frac{\sqrt{5} + \sqrt{3}}{\sqrt{15}}
     \]

     Then multiplying by $\sqrt{15}$ gives $\sqrt{5} + \sqrt{3}$ again.

2. Simplify: $\frac{12x}{3x - 6} \cdot \frac{x^2 - 4}{2x + 4}$

   A. $\frac{x^2 + 12x - 4}{5x - 2}$  B. $2x$  C. $-\frac{4}{3}(x - 1)$  D. $\frac{2x(x - 2)}{x + 2}$  E. $\frac{12x^3 - 48x}{6x^2 - 12}$
• Solution: The answer is B. Recall the difference of squares factorization: $x^2 - a^2 = (x + a)(x - a)$. Here, $x^2 - 4 = (x + 2)(x - 2)$, so
\[
\frac{12x}{3x - 6} \cdot \frac{x^2 - 4}{2x + 4} = \frac{12x(x + 2)(x - 2)}{3(x - 2) \cdot 2(x + 2)}
\]
Cancelling common factors top and bottom yields $2x$.

3. Simplify: $(u^{-5}v^2)^3 \left( \frac{v^2}{u} \right)^{-1}$

A. $u^{-16}v^8$  B. $u^{-14}v^4$  C. $u^{-9}v^3$  D. $(uv)^{-7}$  E. $u^{-4}v^2$

• Solution: The answer is B. Recall the rules for exponents:
\[
a^b \cdot a^c = a^{b+c}, \quad (a^b)^c = a^{bc}, \quad a^{-1} = \frac{1}{a}.
\]
Then
\[
(u^{-5}v^2)^3 \left( \frac{v^2}{u} \right)^{-1} = u^{-15}v^6 \cdot uv^{-2} = u^{-14}v^4.
\]

4. If $f(x) = 5x^2 - 11$, what is $f(a - 2)$?

• Solution: Substitute $a - 2$ for the $x$ in the formula to yield
\[
f(a - 2) = 5(a - 2)^2 - 11 = 5(a^2 - 4a + 4) - 11 = 5a^2 - 20a + 9.
\]

5. Find all values of $x$ satisfying $2(x - 2) > 5$.

• Solution: All real $x > 9/2$ (since $x - 2 > 5/2$, so $x > 5/2 + 2 = 9/2$).

6. Solve for $x$: $2x^2 - x - 6 = 0$ (find all solutions).

• Solution: This quadratic factors as $(2x + 3)(x - 2) = 0$, so $2x + 3 = 0$ or $x - 2 = 0$. Hence $x = -3/2$ or $x = 2$. The equation can also be solved by the quadratic formula:
\[
x = \frac{1 \pm \sqrt{1 + 49}}{4} = 2, -3/2.
\]