Model Solutions for Problem Set 1

MATH 131, Fall 2007

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Section 1.1

2.

a. $f(-4) = -2$ and $g(3) = 4$.

b. $f(x) = g(x)$ at $x = -2, 2$.

c. $f(x) = -1$ at about $x = -3$ and again at $x = 4$.

d. f is decreasing for $0 \leq x \leq 4$, or on the interval $[0, 4]$.

e. The domain of f is the interval $[-4, 4]$. The range of f is the interval $[-2, 3]$.

f. The domain of f is the interval $[-4, 3]$ and the range of g is the interval $[1/2, 4]$.

5. This is not the graph of a function because it does not pass the vertical line test.

6. This is the graph of a function because it does pass the vertical line test. Domain: $[-2, 2]$, Range: $[-1, 2]$.

7. This is the graph of a function because it does pass the vertical line test. Domain: $[-3, 2]$, Range: $[-3, -2) \cup [-1, 3]$.

8. This is not the graph of a function because it does not pass the vertical line test (because of the vertical line segments).

17. The graph should show the length increasing from each Wednesday to the next (roughly linearly). When the grass is cut, the length suddenly jumps back to the cut height and starts growing again.

20.

a. Plot the points and connect them with a smooth curve.

b. When $t = 11$ (hours), the temperature can be estimated as 84.5° (halfway between 81 and 88 degrees).

24. For $f(x) = x^3$,

$$\begin{aligned} \frac{f(a+h)-f(a)}{h} &= \frac{(a+h)^3 - a^3}{h} \\ &= \frac{a^3 + 3a^2h + 3ah^2 + h^3 - a^3}{h} \\ &= 3a^2 + 3ah + h^2 \end{aligned}$$

28. $f(x)$ is undefined when the denominator $x^2+3x+2 = (x+1)(x+2) = 0$. The values $x = -1, -2$ must be excluded, so the domain is the set of real numbers with $x \neq -1, -2$, or $(-\infty, -2) \cup (-2, -1) \cup (-1, +\infty)$.

37. $g(x)$ is undefined when $x - 5 < 0$ hence $x < 5$. The domain is the set $[5, +\infty)$. The graph is obtained from $y = \sqrt{x}$ by shifting 5 units to the right along the x-axis.

42. The domain is the set of all real numbers. The graph should show the portion of the line $y = -\frac{1}{2}x + 3$ (slope $-1/2$) for $x \leq 2$, and the portion of the other line $y = 2x - 5$ (slope 2) for $x > 2$.

52. From the usual area formula, if x, y are the lengths of the sides, $xy = 16$, so $y = \frac{16}{x}$. The perimeter is $P = 2x + 2y$. Substituting for y yields $P = 2x + \frac{32}{x}$.

Section 1.2

2.

a. This is a quotient of polynomials, so *a rational function.*

b. This is an *algebraic function* because of the square root.

c. This is an *exponential function.*

d. This is a *power function.*

e. This is a *polynomial function.*

f. This is a *trigonometric function.*

4.

a $y = 3x$ is the graph $y = G(x)$ (yellow) because that is the only straight line.

b $y = 3^x$ is the graph $y = f(x)$ (blue) because it has the correct shape for an exponential function (and it is the only one that takes only positive values).

c $y = x^3$ is the graph $y = F(x)$ because it has the correct shape for an odd integer power function

d By process of elimination, $y = x^{1/3}$ is the graph $y = g(x)$ (black).

5.

a The family of lines of slope $m = 2$ is given by $y = 2x + b$, where b can be any real number. The graph should show several parallel lines with slope 2.

b $f(2) = 1$ means that the line contains the point $(x, y) = (2, 1)$. By the point-slope form, $y - 1 = m(x - 2)$, so $y = mx - 2m + 1$. The graph should show several lines passing through the point $(2, 1)$, with different slopes.

c The line in both families should have slope $m = 2$ and pass through $(2, 1)$, so by the point-slope form, $y - 1 = 2(x - 2)$, or $y = 2x - 3$.

6. From the given equation $f(-3) = 1$ for all these f. The graphs are all straight lines passing through the point $(x, y) = (-3, 1)$.

8. The first graph is the parabola $y = 2x^2$ shifted 3 units to the right along the x-axis, so $y = 2(x-3)^2$. The second graph is $y = f(x) = Ax^2 + Bx + C$ where from the graph $f(0) = 1$, $f(-2) = 2$, and $f(1) = -2.5$. The first equation says $1 = f(0) = A0^2 + B0 + C$, so $C = 1$. The second and third equations then give:

$$\begin{aligned} 2 = f(-2) &= A(-2)^2 + B(-2) + 1 = 4A - 2B + 1 \\ -2.5 = f(1) &= A(1)^2 + B(1) + 1 = A + B + 1 \end{aligned}$$

or

$$\begin{aligned} 4A - 2B &= 1 \\ A + B &= -\frac{7}{2} \end{aligned}$$

Solving simultaneously yields $A = -1$, $B = -\frac{5}{2}$, so the function is $f(x) = -x^2 - \frac{5}{2}x + 1$.

12.

a. The graph should be the portion of a line sloping down to the right *in the first quadrant*, with vertical axis intercept at $y = 200$, and horizontal axis intercept at $x = 50$.

b. The slope of -4 means that each dollar increase in the price will reduce the number of spaces that will be rented by 4. The y-intercept gives the number of spaces that could be occupied if the rent was free. The x-intercept $x = 50$ gives the price at which no spaces will be rented.

15.

a. $P = 15 + .434d$, where d is the depth in feet and P is the pressure in pounds per square inch at depth d.

b. $100 = 15 + .434d$, when $d = \frac{100-15}{.434} \doteq 195.9$ feet.

18.

a. The slope is $\frac{460-380}{800-480} = \frac{1}{4}$, so the equation is $C - 460 = \frac{1}{4}(d - 800)$, or $C = \frac{1}{4}d + 260$.

b. When $d = 1500$, the model in part a predicts $C = \frac{1}{4}(1500) + 260 = 375 + 260 = 635$ dollars.

c. The slope represents the increase in cost for each additional mile driven.

d. The y-intercept represents the *fixed costs* (car payment, insurance, etc.) that are incurred even if the car is not driven at all.

e. A linear model is reasonable here because there are some costs that are fixed (car payment, insurance, etc.) that are independent of the mileage driven, and others (gas, oil, etc.) that are variable, but proportional to the amount driven. The total cost is the sum of the fixed and variable costs.

Section 1.3

2.

a. The graph is stretched vertically by a factor of 5.

b. The graph is shifted to the right by 5 units along the x-axis.

c. The graph is reflected across the x-axis.

d. The is stretched vertically by a factor of 5 and reflected across the x-axis.

e. The graph is compressed horizontally by a factor of 5.

f. The graph is stretched vertically by a factor of 5, then shifted down by 3 units.

3.

a. This is graph 3, because it is a horizontal shift of $y = f(x)$ by 4 units to the right.

b. This is graph 1, because it is vertical shift by 1 unit up.

c. This is graph 4, because it is a vertical shrinking of $y = f(x)$.

d. This is graph 5, because it is a reflection of $y = f(x)$ across the x-axis, shifted to the left by 4 units.

e This is graph 2, because it is stretched vertically and shifted 6 units to the left.

4. a) Shift 4 units left. b) Shift 4 units up. c) Stretch vertically by 2. d) Compress vertically by 2, reflect across x-axis, then shift up by 3.

6. The graph is stretched vertically by a factor of 2 and shifted 2 units to the right, so we must have

$$y = 2f(x-3) = 2\sqrt{3(x-2)-(x-2)^2} = 2\sqrt{-10+7x-x^2}.$$

10. The graph is obtained by reflecting the parabola $y = x^2$ across the x-axis, then shifting the resulting graph up by 1 unit.

16. The graph is obtained by shifting the graph $y = \frac{1}{x}$ to the right by 4 units. This puts the vertical asymptote at $x = 4$.

30. For each x, the y-coordinate on $y = f(x) + g(x)$ is obtained by adding $f(x)$ and $g(x)$. For example, since it appears $f(-1) = 1$ and $g(-1) = -1$, $f(-1) + g(-1) = 0$. The domain of $f + g$ is the set of x where *both* f, g are defined, so $[-2, 3]$.

32. The domain of f is $[-1, +\infty)$. The domain of g is $(-\infty, 1]$.

- $(f+g)(x) = \sqrt{1+x} + \sqrt{1-x}$. The domain is the set of x that are in both the domain of f and the domain of g, or $[-1, 1]$ (that is all x with $-1 \leq x \leq 1$).
- $(f-g)(x) = \sqrt{1+x} - \sqrt{1-x}$. The domain is the set of x that are in both the domain of f and the domain of g, or $[-1, 1]$ (that is all x with $-1 \leq x \leq 1$).
- $(f \cdot g)(x) = \sqrt{1+x} \cdot \sqrt{1-x} = \sqrt{1-x^2}$. The domain is the set of x that are in both the domain of f and the domain of g, or $[-1, 1]$ (that is all x with $-1 \leq x \leq 1$).
- $(f/g)(x) = \frac{\sqrt{1+x}}{\sqrt{1-x}}$. The domain is the set of x that are in both the domain of f and the domain of g, and where $g(x) \neq 0$: $[-1, 1)$ (that is all x with $-1 \leq x < 1$).

32. The domains of f and g are $(-\infty, +\infty)$. The domain of a composition is the set of x in the domain of the "inside" function such that the value is in the domain of the "outside" function.

- $(f \circ f)(x) = 1 - (1 - x^3)^3$. The domain is the set of all real x.
- $(g \circ g)(x) = \frac{1}{1/x} = x$. The domain is the set of all $x \neq 0$ (since we cannot substitute $x = 0$ into the "inside" $g(x) = \frac{1}{x}$.
- $(f \circ g)(x) = 1 - \frac{1}{x^3}$. The domain is the set of $x \neq 0$.
- $(g \circ f)(x) = \frac{1}{1-x^3}$. The domain is the set of all $x \neq 1$ in the real numbers.

40. The domain of a composition is the set of x in the domain of the "inside" function such that the value is in the domain of the "outside" function.

- $(f \circ f)(x) = \sqrt{2\sqrt{2x+3}+3}$. The domain is the set of all real $x \geq -3/2$.
- $(g \circ g)(x) = (x^2 + 1)^2 + 1$. The domain is the set of all real x.
- $(f \circ g)(x) = \sqrt{2(x^2+1)+3} = \sqrt{2x^2+5}$. The domain is the set of all real x.
- $(g \circ f)(x) = (\sqrt{2x+3})^2 + 1 = 2x + 4$. The domain is the set of all $x \geq -3/2$ in the real numbers, since we must be able to substitute x into the inside $f(x)$.

42.

$$(f \circ g \circ h)(x) = \frac{2}{\cos(\sqrt{x+3}) + 1}$$

44. $F(x) = \sin(\sqrt{x}) = (f \circ g)(x)$ for $f(x) = \sin(x)$ and $g(x) = \sqrt{x}$.

50. a) $f(g(1)) = f(6) = 5$. b) $g(f(1)) = g(3) = 2$. c) $f(f(1)) = f(3) = 4$. d) $g(g(1)) = g(6) = 3$. e) $(g \circ f)(3) = g(f(3)) = g(4) = 1$. f) $(f \circ g)(6) = f(g(6)) = f(3) = 4$.

54. a) The radius is increasing at the constant rate of 2 cm/sec. After t seconds, it is given by $r = 2t$. b) From the volumn formula for a sphere, $V = \frac{4}{3}\pi r^3$, so $(V \circ r)(t) = \frac{4}{3}\pi(2t)^3 = \frac{32}{3}\pi t^3$. This gives the volume of the balloon as a function of time.

55. a) The positions of the ship when it is closest to the lighthouse, the position at a later time, and the lighthouse form a right triangle with sides $d, 6$ and hypotenuse s. By the Pythagorean Theorem, $s = \sqrt{d^2 + 36}$. b) $d = 30t$. c) $s = \sqrt{(30t)^2 + 36} = \sqrt{900t^2 + 36}$. This function gives the distance to the lighthouse as a function of time.