MATH 131, section 1 – Practice Questions for Exam 2
October 17, 2007

1. All parts of this question refer to the parametric curve $x = 3 \cos(2t)$, $y = 5 \sin(2t)$.

(a) Eliminate the parameter $t$ and find a Cartesian equation for this curve.

(b) What portion of the curve is traced out for $0 \leq t \leq \frac{\pi}{2}$, and in which direction is the curve being traced?

(c) What would change in your answer to part b if the curve above was replaced by $x = 3 \cos(-2t)$, $y = 5 \sin(-2t)$

2. Use the sum, product, and/or quotient rules to compute the following derivatives. You may use any correct method, but must show work and simplify your answers for full credit.

(a) $\frac{d}{dx} \left(5x \sqrt{x} - \frac{2}{x^3} + 11x - 4\right)$

(b) $\frac{d}{dt}(t^2 e^t)$

(c) $\frac{d}{dz} \left(\frac{z^2 - 2z + 4}{z^2 + 1}\right)$

(d) $\frac{d}{dx} \left(\frac{\pi^2 + \tan(e^x) - 2x^e}{4}\right)$

3. The graph of a function $f$ is shown below with several points marked. Find all the marked points at which the following are true, and give explanations for your answers.

- $f$ is discontinuous.
- $f$ is continuous, but the graph of $f$ has a vertical tangent line.
- $f$ is continuous, but the graph of $f$ has no tangent line.
4. Compute the indicated limits. Show all work for full credit.

(a) \[ \lim_{x \to 1} \frac{3x^2 - 5x - 2}{x^2 - 4x + 4} \]
(b) \[ \lim_{x \to 2} \frac{3x^2 - 5x - 2}{x^2 - 4x + 4} \]
(c) \[ \lim_{x \to \infty} \frac{3x^2 - 5x - 2}{x^2 - 4x + 4} \]
(d) \[ \lim_{x \to 2^+} \frac{|x - 2|}{x^2 - 5x + 6} \]

5. Let \( f(x) = x^3 - x^2 \).

(a) Find all intervals on which \( f \) is **decreasing**.
(b) Find all intervals on which \( f \) is **concave up**.
(c) Find all intervals on which \( f \) is both **increasing** and **concave down**.

6. Do not use the differentiation rules from Chapter 3 in this question.

(a) State the limit definition of the derivative \( f'(x) \).
(b) Use the definition to compute the derivative function of \( f(x) = \frac{1}{3x} \).
(c) Find the equation of the line tangent to the graph \( y = \frac{1}{3x} \) at \( x = 2 \).

7. The total cost (in $) of repaying a car loan at interest rate of \( r \)\% per year is \( C = f(r) \).

(a) What is the meaning of the statement \( f(7) = 20000 \)?
(b) What is the meaning of the statement \( f'(7) = 3000 \)? What are the units of \( f'(7) \)?