Some possibly useful formulas:
\[
\sin^2 \theta + \cos^2 \theta = 1, \quad \tan^2 \theta + 1 = \sec^2 \theta, \quad \sin(2\theta) = 2 \sin \theta \cos \theta.
\]

(5) A) Integrate:
\[
\int \frac{x^3}{\sqrt{9 - x^2}} \, dx
\]
Because of the $9 - x^2$ under the radical sign, we use the sin substitution, letting $x = 3 \sin \theta$. Then $dx = 3 \cos \theta \, d\theta$ and we proceed as follows:
\[
\int \frac{x^3}{\sqrt{9 - x^2}} \, dx = \int \frac{(3 \sin \theta)^3}{\sqrt{9 - (3 \sin \theta)^2}} \, 3 \cos \theta \, d\theta
\]
\[
= \int \frac{27 \sin^3 \theta \cdot 3 \cos \theta}{\sqrt{9(1 - \sin^2 \theta)}} \, d\theta
\]
\[
= \int \frac{27 \sin^3 \theta \cdot 3 \cos \theta}{\sqrt{9 \cos^2 \theta}} \, d\theta \quad \text{(by the basic identity)}
\]
\[
= \int \frac{27 \sin^3 \theta \cdot 3 \cos \theta}{3 \cos \theta} \, d\theta
\]
\[
= 27 \int \sin^3 \theta \, d\theta \quad \text{(after cancelling)}
\]
\[
= 27 \int \sin^2 \theta \cdot \sin \theta \, d\theta \quad \text{(odd power strategy)}
\]
\[
= 27 \int (1 - \cos^2 \theta) \sin \theta \, d\theta
\]
\[
= 27 \left( \int \sin \theta \, d\theta - \int \cos^2 \theta \sin \theta \, d\theta \right) \quad \text{(multiply out and split)}
\]
\[
= 27 \left( - \cos \theta + \frac{1}{3} \cos^3 \theta \right) + C
\]
The last line comes from applying the $u$-substitution $u = \cos \theta$ to the second integral (note that $du = - \sin \theta \, d\theta$). Now we convert back to functions of $x$ from the substitution $x = 3 \sin \theta$. The reference triangle has opposite side $x$, hypotenuse $3$ and adjacent side $\sqrt{9 - x^2}$. Hence $\cos \theta = \sqrt{9 - x^2}/3$, and the integral equals
\[
= -9 \sqrt{9 - x^2} + \frac{1}{3} (9 - x^2)^{3/2} + C.
\]
(5) B) Integrate:
\[
\int \frac{x^4 + 2x + 1}{x^3 + x} \, dx
\]

Since the degree of the top is larger, we have to divide the denominator into the numerator. The quotient is \(x\) and the remainder is \(x^4 + 2x + 1 - x(x^3 + x) = -x^2 + 2x + 1\). Therefore,
\[
\frac{x^4 + 2x + 1}{x^3 + x} = x + \frac{-x^2 + 2x + 1}{x^3 + x} = x + \frac{-x^2 + 2x + 1}{x(x^2 + 1)}.
\]

We split the second term up into partial fractions like this:

\[
\frac{-x^2 + 2x + 1}{x(x^2 + 1)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 1}, \quad \text{(so after clearing denoms)}
\]

\[-x^2 + 2x + 1 = A(x^2 + 1) + x(Bx + C)\]

\[-x^2 + 2x + 1 = (A + B)x^2 + Cx + A\]

By comparing coefficients \(A = 1, C = 2\) and \(A + B = -1\). Hence \(B = -2\). This gives
\[
x + \frac{-x^2 + 2x + 1}{x(x^2 + 1)} = x + \frac{1}{x} + \frac{-2x + 2}{x^2 + 1} \quad \text{(so)}
\]

\[
\int x + \frac{-x^2 + 2x + 1}{x(x^2 + 1)} \, dx = \int x + \frac{1}{x} + \frac{-2x + 2}{x^2 + 1} \, dx
\]

\[
= \frac{x^2}{2} + \ln|x| + \int \frac{-2x}{x^2 + 1} \, dx + \int \frac{2}{x^2 + 1} \, dx
\]

\[
= \frac{x^2}{2} + \ln|x| - \ln(x^2 + 1) + 2 \tan^{-1}(x) + C
\]

The third term here comes from the integral \(\int \frac{-2x}{x^2 + 1} \, dx\), which is \(-\int \frac{du}{u}\) for \(u = x^2 + 1\). The last term is the basic inverse tangent integral.