MATH 131, section 1 – Calculus for Physical and Life Sciences 1
Lab Day 2 – Parametric Curves
Writeups due: Monday, September 24 in class

Please work in your lab groups of two from the first lab day.

Background and Goals

Suppose a particle is moving along a curve in the xy-plane. Then, at a time t the particle will be at a point on the curve with coordinates (x, y), where x and y are two functions of t, $x = f(t)$ and $y = g(t)$. The two equations for x and y are called *parametric equations* with *parameter* t. Today's lab will show you several additional examples of this idea and give you a way to visualize how curves are traced out this way.

Lab Questions

A. Consider a particle whose coordinates at time t are $x = 2t + 1$ and $y = 3t + 2$.

1) Find the coordinates (x, y) of the particle at times $t = -2, -1, 0, 1, 2$ and plot the points (x, y) to guess the curve, using a Maple command of the following format:

```
plot( -your point list- , style=point,symbol=circle);
```

   where `-your point list-` is the list of (x, y) coordinates you generated, in the form `[[x1,y1],[x2,y2],[x3,y3],[x4,y4],[x5,y5]]`. What curve do you expect these points lie on?

2) Parametric curves can be plotted in Maple. Use this feature to check your answer above. Type

```
plot([2*t+1,3*t+2,t=-2..2]);
```

   Of course, you may also change the interval for t.

3) To see how the particle traverses the curve, we will use the animation feature of Maple. Type

```
with(plots): animatecurve([2*t+1,3*t+2,t=-10..10],frames=30);
```

   After entering the command you will see a set of axes. Click on the picture. At the top of the screen you will see a button labelled ▷. Click on it. You will see the curve being traced. You can also look at the curve being traced one piece at a time by clicking repeatedly the button to the right of the ▷ button.

4) What happens if you replace t by $-t$ in the parametric equations?

B. Now we will study the motion of a particle with coordinates given by

$$x = 4\cos(t) \qquad y = 3\sin(t)$$

1) First reason without Maple's help (i.e. use pen and paper). What is the shape of the curve traversed by the particle? How long does it take the particle to traverse the curve once? Use this interval for t to plot the parameterized curve. (The Maple syntax for π is `Pi`.) Then animate the plot. In what direction is the curve being traversed?

2) What happens if you change t to $-t$ in the parametric equations?

3) What if you change t to $2t$?

4) What if you change t to $t/100$?

5) In each case, how long does it take the particle to go once around the curve?

6) What if you just change the interval for t (say, you have $0 \leq t \leq 10\pi$)?

C. For each $n = 2, 3, 4, 5$, animate the motion of an object moving along the curve

$$x = \cos(t) \qquad y = \sin(nt), \qquad 0 \leq t \leq 2\pi.$$

Describe the effect of varying n on the curve above. (The pattern described by these curves is called a *Lissajous figure*.)